An Optimization Approach for Real Time Evacuation Reroute Planning

Gino J. Lim*and M. Reza Baharnemati*and Seon Jin Kim*

November 16, 2015

Abstract

This paper addresses evacuation route management in the case of incidents arising during an evacuation and aims to minimize further delays they may cause. An evacuation reroute planning approach is developed for decision makers to utilize alternative routes in real time for the evacuees whose evacuation paths are affected by an incident during the evacuation process. Assuming that real time traffic information is available, a preprocessing algorithm is performed to update the evacuation networks. A multi-commodity network flow optimization model is then utilized to develop alternative paths and corresponding flow rates. Due to the underlying optimization model being a mixed integer nonlinear programming formulation, a linear reformulation of the model is developed to improve the computational performance. In the numerical results, the performance of the proposed decision making tool is tested using a case study.

Key words: Real time evacuation route planning, Network flows, Evacuation

1 Introduction

Regional evacuation should be performed in case of terrorist attacks, man-made disasters and natural disasters. Devising an effective evacuation plan requires an exact description of the underlying transportation infrastructure. This information of the transportation infrastructure is translated into an evacuation network consisting of roads, intersections and exit points. In the evacuation

*Department of Industrial Engineering, University of Houston, Corresponding author: ginolim@uh.edu.

network, it is critical to keep precise information of the transit time and capacity of the roads. However, this information has an uncertain nature and it can change due to occurrence of unexpected incidents. A car accident, a bridge collapse, or any other type of incident during evacuation can cause severe traffic congestion. Evacuation delays are often caused by traffic congestion affecting the partial or total closure of roads for long or short periods of time. These delays may require an alternative to the original evacuation route plan. In general, a minor incident may not trigger rerouting because it probably is better to stick to the original plan and wait until the incident is cleared. However, a severe incident can create an unreasonable amount of traffic congestion which may require alternative paths to be utilized to evacuate the outbound flows and minimize evacuation clearance time.

An efficient and complete evacuation route plan consists of two phases: before and after the arrival of an anticipated disaster. The first phase of evacuation route planning takes place before the arrival of an imminent disaster. The goal of this phase is to evacuate as many people as possible within a minimal evacuation clearance time (Baharnemati and Lim (2011)). The plan should provide the evacuation paths, flow rates, and schedules for all the evacuees leaving the impacted areas moving toward the safe destinations. The second phase of route planning is to monitor the progress of evacuation and make adjustments on the evacuation routes if necessary. Rerouting methods are utilized during the second phase which provides new paths for evacuees whose evacuation paths are affected by an incident. The process of generating the alternative paths should be quick and the field officers should receive the new information as soon as it is available in order to avoid further interruption in the evacuation process. For example, if a road accident occurs during evacuation, the incident-related information is gathered and processed. If it affects one of the current evacuation paths, an alternative path is generated to mitigate the consequence of the incident.

For this paper, an optimization approach for real-time hurricane evacuation reroute planning is considered to generate alternative paths and flow rates for evacuees stranded on roads due to incidents that require a significant amount of time to clear. The incident-related information that is assumed to be available at the time of reroute planning includes incident time, location, and severity of congestion, to name a few. We also assume that the incident end time can be estimated and readily available. The proposed decision making approach consists of two main elements: network preprocessing algorithm and multi-commodity network flow optimization model. The purpose of the preprocessing algorithm is to update the evacuation networks at the time that an incident occurs to determine the locations of evacuees on the network. To generate a set of alternative paths and flow rates, it is necessary to update the evacuation dynamic network such that the marginal (or residual) arc capacities are adjusted based on the flow rates of the evacuation paths up to the time of the incident. Therefore, through the preprocessing algorithm, the evacuation paths are categorized into affected and unaffected paths by an incident. For the unaffected paths, the capacity of associated arcs is decreased by the path flow rate for the entire time that the path is scheduled to be utilized. For the affected path, the arc capacities are updated up to the incident start time. This is because our aim is to reroute evacuees on the affected path from the starting point of the incident (the first immediate intersection before the incident) to the destination that was originally identified. For each affected path, a commodity is defined for those that are affected by the incident.

Four features are considered for each commodity:1) source node of commodity, which is the starting point of the incident; 2) destination node of commodity, which is the destination node of the evacuees; 3) commodity schedule, which is the time interval between A and B where A is the maximum between the time when the head of the evacuees reach the incident starting point and the start time of the incident, and B is the minimum between the time that the tail of evacuees reach the incident starting point and the incident end time; and 4) supply of commodity, which is the number of evacuees that have arrived at the incident starting point at the commodity schedule. After applying the preprocessing algorithm and defining the commodities, a multi-commodity network flow optimization model is utilized to find the alternative paths and their corresponding flow rates for the commodities.

This paper focuses on the second phase of hurricane evacuation routing and assumes that decision makers have information on the evacuation route plan (evacuation paths, flow rates, schedule, and horizon times) and information related to the incident (start time of the incident and estimated end time of incident) in advance.

As in Lim et al. (2012), travel time is estimated based on severe rush hour traffic conditions to be conservative. Hence, the actual traffic conditions may likely be better than the plan generated by this approach. There are two different approaches to assign flow rates to the paths. Flow rates can be constant or variable over the time horizon. In variable flow rates, the number of evacuees that are leaving source nodes can change in each time period. A drawback of the variable flow rate plan, however, is that it could be confusing and can be difficult to implement in practice. For instance, there can be a zero flow rate at certain time periods at which the evacuees need to be stopped from using the evacuation paths. In such a case, it is required to employ more resources to control and manage the evacuation process. Hence, a variable flow rate can be considered impractical. Unlike the variable flow rate scheme, a constant flow rate approach is more practical. From the managerial point of view, having a fixed flow rate makes the evacuation plan easy to understand and execute. Therefore, a constant evacuation flow rate is assumed for each alternative path over the planning horizon to help operation managers in flow control and management. A fixed flow rate for evacuation paths makes the decision making tool more applicable and less confusing as noted in Baharnemati and Lim (2011).

The scope of this paper is limited to developing a rerouting plan when an incident occurs in the evacuation network. It can certainly be expanded to instances with multiple incidents. However, having multiple incidents on the same path at the same time is rare in practice. The rest of the paper is organized as follows. It will begin by describing a short review of literature relevant to the work in Section 2. A detailed description of the preprocessing algorithm is explained in Section 3. Different scenarios having one incident are discussed in Section 3.1. In Section 4, the multi-commodity network flow optimization model is presented. Computational results are conducted to examine the performance of proposed decision making tool in Section 5. This paper is concluded with a short summary in Section 6.

2 A brief literature review

There are three stages in evacuation planning and management: preparedness, response, and recovery. One can also add mitigation as another important component to this category. Numerous papers have been published address each of these categories (Cova and Johnson (2003); Galindo and Batta (2013); Lim et al. (2015); Ng and Waller (2010); Yao et al. (2009); Zheng and Chiu (2011)). The discussion of this section has been limited to the *response* stage as it is the primary subject of this paper.

The first step of the evacuation route planning problem is to find the initial optimal path, flow, and schedule for each source node to its destination node with various objectives and constraints (Baharnemati and Lim (2011), Lim et al. (2012), and Rungta et al. (2012)). However, the static plans can be severely affected if a major accident occurs in the evacuation network, which may require rerouting of evacuees depending on the severity of the incident. Several researchers have addressed issues of finding optimal paths that can be useful in real time rerouting. Kok et al. (2012) used modified Dijkstra algorithm and heuristic dynamic programming to find the optimal paths avoiding traffic congestion. Akgün et al. (2007) developed a heuristic approach to deal with routing under a weather condition. Desai and Lim (2013*a*) developed a stochastic dynamic model for hazmat vehicle rerouting in real time. They presented a network reoptimization framework based on information of links for rerouting hazmat vehicles (see also Desai and Lim (2013*b*)).

According to the literature, little attention was given to real-time evacuation reroute planning for emergency evacuation circumstance. Therefore, based on the optimal static evacuation plan (Lim et al. (2012)), the aim is to develop an optimization approach to find an alternative path if the path was affected by a severe incident.

3 Problem description and network preprocessing

The problem in this paper deals with rerouting evacuees on the road due to an incident that occurred during the evacuation process. Due to the incident occurring during an evacuation, the state of the evacuation network may change and the original evacuation plan may no longer be valid. Therefore, the evacuation network needs to be updated to reflect the situation of the evacuees and the arc capacities when the incident occurs. Having the most up-to-date information on the evacuation network will help generate alternative paths with associated flow rates. This section analyzes different situations that can arise on the affected paths if an incident occurs (Section 3.1), followed by the network preprocessing algorithm (Section 3.2).

3.1 Scenario Analysis associated with an incident on the evacuation network

Assume that an incident occurred on arc (i, j) during time interval $TP_1 = [t_1, t_2]$ in the evacuation network, i.e. the incident starts at time t_1 and ends at time t_2. Node i is considered as incident starting point and node j as incident end point. Suppose that path p originates from source node s and terminates at destination node d via impacted arc (i, j) (see Figure 1). Let tr_1 be the travel time from source node s to node i on path p.

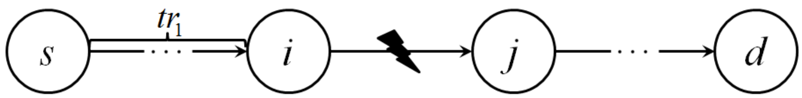

Figure 1: A sample path affected by an incident

Let LT_0 be the start time of departure leaving source node s and LT_n be the last time of departure from the same node. Then, the first time that the evacuees reach node i via path p is $t'_1 = LT_0 + tr_1$ and the last time will be $t'_2 = LT_n + tr_1$. Therefore, the time interval that evacuees arrive at node i via path p is $TP_2 = [t'_1, t'_2]$. This gives six different travel scenarios as seen in Figure 2.

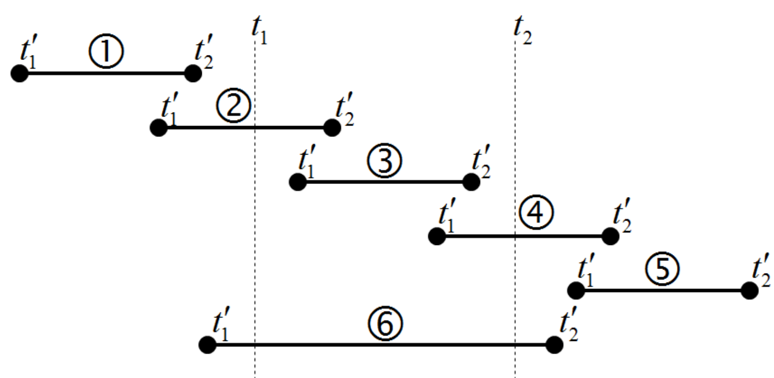

Figure 2: Different cases for one incident in the evacuation network

Cases 1 and 5 are not affected by the incident because all evacuees either had passed the incident starting point (i) before the incident occurred, or will not reach the incident starting point (i) until the incident arc is cleared. Therefore, evacuees on path p do not need to be rerouted. For the rest of the cases, TP_2 has a partial or a full overlap with TP_1. Consequently, path p needs to be revised to reroute the affected evacuees. For example, the path is partially affected for cases 2 and 4; hence, an alternative path needs to be developed from time t_1 to t'_2 and t'_1 to t_2, respectively. However, in Case 3, path p is affected in its entire schedule and needs to be replaced with a new path from time t'_1 to t'_2. Finally, we need to provide an alternative path from time t_1 to t_2 for Case 6.

3.2 Network preprocessing algorithm

The state of evacuees changes over time during evacuation. If an incident occurs on the evacuation network, the original evacuation path may no longer be valid and an alternative path is necessary to reroute the affected evacuees to a safe destination. A network preprocessing algorithm (NPA) is proposed to obtain the most up-to-date evacuation network for developing an alternative path. The outputs of NPA are the revised static evacuation network, dynamic evacuation network and a set of commodities. These outputs are the inputs for the optimization model described in Section 4.

Algorithm 1 Network preprocessing algorithm

Inputs:
  A static network $\mathscr{G}$ consisting of a set of nodes $\mathscr{N}$ and a set of arcs $\mathscr{A}$.
  An evacuation routing plan (i.e., evacuation paths, flow rates, and schedule for each source node).
  Impacted arc information (i.e., tail, head, start time, and end time).
Step 1 - Initialization:
  Construct a dynamic evacuation network based on the static evacuation network.
  Define a set $\mathscr{P}$ which includes all the evacuation paths.
  Define an empty set $\mathscr{K}$ which will include commodities.
Step 2 - Paths classification:
for all path $p \in \mathscr{P}$ **do**
  if path p contains no impacted arcs **then**
    Run Procedure 2 based on the flow rate (f_p) and schedule of path p ($[LT_0, LT_n]$).
    Reduce the capacity of destination node of path p by $f_p(LT_n - LT_0 + 1)$.
  else if path p contains one impacted arc **then**
    Run Procedure 1.
  end if
end for
Step 3 - Removing the impacted arc:
Remove the impacted arc from the static network as well as the dynamic network.

The network preprocessing algorithm consists of three steps. In the first step, a dynamic evacuation network is constructed based on three sets of inputs: 1) a static evacuation network; 2) an initial evacuation route plan (i.e. evacuation paths, flow rates, and schedules for each source node); and 3) impacted arcs information (i.e. tail, head, start time, and end time). Then, two sets ($\mathscr{P}$ and $\mathscr{K}$) are initialized; $\mathscr{P}$ contains all evacuation paths and set $\mathscr{K}$ as a set of commodities; $\mathscr{K}$ is initially an empty set and its entries will be added to the set in Step 2 based on the condition of the evacuation paths.

In Step 2, the evacuation paths are categorized into two main groups.

The first group consists of a set of paths that are not affected by an incident and the second group is for the paths affected by the incident. These two sets of groups are mutually exclusive. Two procedures are developed to update changes on the evacuation network: identification of the path containing the impacted arc (Procedure 1) and dynamic network update procedure (Procedure 2). These procedures are explained in detail later. As soon as evacuees move along the network toward their destinations, the residual network capacities need to be frequently updated so that alternative paths can be generated based on the residual network capacity. Procedure 2 is developed for the network update based on the current flow rate of the path (f_p) and its schedule ($[LT_0, LT_n]$). As a result, the destination node capacity of the path is reduced accordingly, i.e. $f_p \cdot (LT_n - LT_0 + 1)$. At the final step of the preprocessing, the impacted arc is removed from the static network as well as the dynamic network to ensure that it will not appear in the alternative paths.

Procedure 1 Preprocessing: identification of an impacted arc

Calculate tr_1 and determine $TP_2 = [t'_1, t'_2]$.
Find the row (r) in Table 1 corresponding to $TP_1$and TP_2.
Run Procedure 2 based on f_p and the updating schedule of row r.
Reduce the destination node capacity of path p based on f_p and the updating schedule of row r.
Create a new commodity based on f_p and the commodity schedule of row r and add it to set $\mathscr{K}$.

The primary goal of Procedure 1 is to identify the evacuation paths that are affected by an incident and that are required to be rerouted. First, the travel time (tr_1) from the source node of an evacuation path to the incident starting point is calculated and time period $TP_2 = [t'_1, t'_2]$ is updated. Then, Table 1 is used to determine the scenario that matches the current situation for TP_1 and TP_2. The first column of Table 1 lists all possible scenarios due to a single incident and the second column shows conditions that govern each scenario based on TP_1 and TP_2. For each scenario, the third column shows the time period that an alternative path originating from node i is needed and the last column is the time period that the dynamic evacuation network should be updated based on f_p.

If path p belongs to Case 1 or Case 5, it does not need to be rerouted. However, based on the flow rate of path p and its schedule, the dynamic evacuation network is updated by using Procedure 2. The capacity of destination node of path p is decreased by $f_p(LT_n - LT_0 + 1)$. If path p is classified

Table 1: All possible scenarios having one incident in an evacuation path

Case No.	Condition	Commodity schedule	Updating schedule
1	$t'_2 < t_1$	–	$[LT_0, LT_n]$
2	$t'_1 < t_1 \leq t'_2 \leq t_2$	$[t_1, t'_2]$	$[LT_0, \mu_1]$
3	$t_1 \leq t'_1$ & $t'_2 \leq t_2$	$[t'_1, t'_2]$	–
4	$t_1 \leq t'_1 \leq t_2 < t'_2$	$[t'_1, t_2]$	$[\mu_2, LT_n]$
5	$t_2 < t'_1$	–	$[LT_0, LT_1]$
6	$t'_1 < t_1$ & $t_2 < t'_2$	$[t_1, t_2]$	$[LT_0, \mu_1]$&$[\mu_2, LT_n]$

$\mu_1 = t_1 - tr_1 - 1 \quad \mu_2 = t_2 - tr_1 + 1$

as Case 3, a new evacuation path is required to replace the old one. The alternative path will begin before the incident start point and will end at the same destination node of path p. It is assumed that evacuees will begin the evacuation process according to the schedule via path p and as soon as they reach the incident start point, they will be rerouted to an alternative path. The amount of supply that should be rerouted from the incident start point is $f_p(LT_n - LT_0 + 1)$. Therefore, for path p, a new commodity is defined with schedule $[t'_1, t'_2]$ and supply $f_p(LT_n - LT_0 + 1)$ and is added to set $\mathscr{K}$. If path p is partially affected as in Case 2, Case 4, or Case 6, the dynamic network is first updated using Procedure 2 based on f_p and the corresponding schedule shown in Table 1. The updating schedules for Cases 2 and 4 are $[LT_0, t_1 - tr_1 - 1]$ and $[t_2 - tr_1 + 1, LT_n]$, respectively, while $[LT_0, t_1 - tr_1 - 1]$ and $[t_2 - tr_1 + 1, LT_n]$ are the ones for Case 6. After updating the dynamic network, the destination node capacity of path p is decreased by $f_p(t_1 - tr_1 - LT_0)$, $f_p(LT_n - t_2 + tr_1)$, and $f_p(LT_n - LT_0 + t_1 - t_2)$ for cases 2, 4, and 6, respectively. Next, a new commodity for path p is created with schedule $[t_1, LT_n + tr_1]$ and supply $f_p(LT_n + tr_1 - t_1 + 1)$ for Case 2, with schedule $[LT_0 + tr_1, t_2]$ and supply $f_p(t_2 - LT_0 + tr_1 + 1)$ for Case 4, and with schedule $[t_1, t_2]$ and supply $f_p(t_2 - t_1 + 1)$ for Case 6, and it is added to set $\mathscr{K}$.

Procedure 2 Dynamic network updating procedure

Inputs: An evacuation path p, its flow rate (f_p), and the corresponding schedule $[\eta_1, \eta_2]$.
Procedure:
$\kappa_1 \leftarrow \eta_1$
while $\kappa_1 \leq \eta_2$ **do**
  $\Lambda \leftarrow$ the first arc of path p.
  $\kappa_2 \leftarrow \kappa_1$
  loop
    $\mathscr{U}_\Lambda^{\kappa_2} \leftarrow \mathscr{U}_\Lambda^{\kappa_2} - f_p$
    $\kappa_2 \leftarrow \kappa_2 + t_\Lambda$
    if $\Lambda \neq$ the last arc of path p **then**
      $\Lambda \leftarrow$ the next arc of path p.
    else
      Break loop!
    end if
  end loop
  $\kappa_1 \leftarrow \kappa_1 + 1$
end while

The procedure of updating the dynamic evacuation network requires two inputs: the flow rate of path p (f_p) and the time period for which path p is utilized, ($[\eta_1, \eta_2]$). The capacity of arcs associated with path p should be decreased by f_p. Parameter κ_1 is used to advance time from η_1 to η_2. At each time period κ_1, all arcs in path p are evaluated and the arc capacity is decreased by f_p at the time the arc is reached. Parameter κ_2 is used to control the arrival time at an arc, which is initialized in each time period and is set to κ_1 (i.e., $\kappa_2 = \kappa_1$). Every time an evacuee departs an arc, κ_2 is progressed by the arc transit time (i.e., $\kappa_2 = \kappa_2 + t_\Lambda$) and the arc capacity is modified (i.e., $\mathscr{U}_\Lambda^{\kappa_2} = \mathscr{U}_\Lambda^{\kappa_2} - f_p$). As soon as we reach the last arc in path p, κ_1 is incremented by one time unit and the procedure continues until the stopping criterion is met.

4 Problem formulation

After applying the preprocessing algorithm on the static evacuation network and the dynamic evacuation network, a multi-commodity network flow optimization model is developed to find alternative paths and the corresponding flow rates. As mentioned previously, to make the alternative paths less confusing to the evacuees, the evacuees are rerouted from the incident start points and to the same destination nodes as originally planned. In doing so, one path for each commodity is considered with a fixed flow rate. Having more than one path for each commodity makes the evacuation route plan more difficult to implement.

The following parameters are introduced to be used in the mathematical optimization formulation. The static evacuation network $\mathcal{D} = (\mathscr{N}, \mathscr{A})$ is composed of a set of nodes $\mathscr{N}$ and a set of arcs $\mathscr{A}$. For each arc $(i,j) \in \mathscr{A}$, let $t(i,j)$ be the arc transit time and $\mathscr{U}^t(i,j)$ be the arc capacity at time t. Nodes in the network are categorized into commodity source nodes ($\mathscr{N}_s$), intermediate nodes, and destination nodes ($\mathscr{N}_d$). Let $\mathcal{S}_k$ be the supply of commodity $k \in \mathscr{K}$ and $\mathscr{C}_j$ be the capacity of destination node $j \in \mathscr{N}_d$. It is assumed that there are T time periods $\{0, 1, \cdots, T-1\}$ to complete evacuation of supplies (i.e. evacuees) from commodity source nodes to destination nodes. The source node of commodity k is defined as $\mathbb{O}_k$ and $\mathbb{D}_k$ as the destination node of commodity k for each commodity $k \in \mathscr{K}$.

The model has two decision variables. A binary variable $y_k(i,j)$ takes value 1 if the path of commodity k uses arc (i,j) and 0, otherwise; and f_k is a positive integer variable for path flow rate of commodity k. An auxiliary variable β_k^1 is also introduced to this model as a slack variable associated with each commodity k, $\forall k \in \mathscr{K}$. Typically, evacuation planning optimization is focused on minimizing the evacuation time. However, it has been defined that the objective of our model is to maximize the total number of transported evacuees from the commodity source nodes to the destination nodes over clearance time T. This goal can be interpreted as having the minimum number of remaining evacuees in the commodity source nodes at the end of the time horizon. Therefore, the objective function is set to minimize the total remaining evacuees in the commodity source nodes at time T. The proposed multi-commodity network flow model of the problem can be stated as follow.

$$\text{Minimize} \quad \sum_{k \in \mathscr{K}} \beta_k^1, \tag{1}$$

$$\text{Subject to:} \quad \sum_{j|(i,j)\in\mathscr{A}} y_k(i,j) - \sum_{j|(j,i)\in\mathscr{A}} y_k(j,i) = 1, \qquad \forall k \in \mathscr{K}, \forall i = \mathbb{O}_k, \tag{2}$$

$$\sum_{j|(i,j)\in\mathscr{A}} y_k(i,j) - \sum_{j|(j,i)\in\mathscr{A}} y_k(j,i) = 0, \qquad \forall k \in \mathscr{K}, \forall i \in \mathscr{N}, i \neq \mathbb{O}_k \text{ and } \mathbb{D}_k, \tag{3}$$

$$\sum_{j|(i,j)\in\mathscr{A}} y_k(i,j) - \sum_{j|(j,i)\in\mathscr{A}} y_k(j,i) = -1, \qquad \forall k \in \mathscr{K}, \forall i = \mathbb{D}_k, \tag{4}$$

$$\sum_{k \in \mathscr{K}} f_k y_k(i,j) \leq \min_{t \in \mathscr{T}} \mathscr{U}^t(i,j), \qquad \forall (i,j) \in \mathscr{A}, \tag{5}$$

$$(\Theta_k^2 - \Theta_k^1 + 1)f_k + \beta_k^1 \geq \mathcal{S}_k, \qquad k \in \mathscr{K} \tag{6}$$

$$\sum_{k \in \mathscr{K} | \mathcal{D}_k = j} (\Theta_k^2 - \Theta_k^1 + 1)f_k \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_d, \tag{7}$$

$$y_k(i,j) \in \{0,1\}, \qquad \forall k \in \mathscr{K}, \forall (i,j) \in \mathscr{A}, \tag{8}$$

$$f_p, \beta_k^1 \in \mathbb{Z}_+, \qquad \forall k \in \mathscr{K}, \tag{9}$$

Constraints (2), (3), and (4) are general network flow optimization constraints to generate a set of optimal paths. Constraint (5) is to make sure that the total flow on arc (i, j) in each time period does not exceed the arc capacity. Since f_k is fixed over the time horizon, the total flow on arc (i, j) is compared to the minimum value of the arc capacity over the time. The total number of times that a path can be utilized for commodity k over the commodity schedule is $\Theta_k^2 - \Theta_k^1 + 1$. Therefore, the total assigned flow for a commodity k over $[\Theta_k^1, \Theta_k^2]$ through a path can be calculated by $(\Theta_k^2 - \Theta_k^1 + 1)f_k$. In addition, since the commodity schedule and the corresponding path flow rate are integers, the total assigned flow to the path of commodity k over its schedule could be less or greater than the supply of commodity k. To guarantee an integer feasible solution for the model a slack variable (i.e. β_k^1) is introduced that accounts for the difference from $\mathcal{S}_k$, i.e., $\beta_k^1 > 0$ if $(\Theta_k^2 - \Theta_k^1 + 1)f_k < \mathcal{S}_k$, $\beta_k^1 = 0$, Otherwise. Hence, Constraint (6) shows that the total assigned flow to the path of commodity k over $[\Theta_k^1, \Theta_k^2]$ plus β_k^1 should be greater than $\mathcal{S}_k$. Here, β_k^1 can be interpreted as remaining supply of commodity k. Consequently, the objective function is to maximize the total transported supplies by minimizing the total remaining supplies. A destination node j cannot accept more flow than its capacity. Constraint (7) limits the total incoming flow to node j to its capacity. This is a mixed integer nonlinear programming model that can be difficult to solve. Therefore, it is proposed to linearize the model using Proposition 1.

Proposition 1. *Suppose that λ is a binary variable and γ is a positive integer variable with an upper bound of $\mathscr{M}$. Then, $\lambda\gamma$ can be linearized by defining μ as a new positive real variable that is equal to $\lambda\gamma$ as follows:*

$$\gamma + \mathscr{M}(\lambda - 1) \leq \mu \leq \mathscr{M}\lambda, \tag{10}$$

$$0 \leq \mu \leq \gamma, \tag{11}$$

Proof. See (Glover, 1975) for details. □

Let a new variable be defined as $w_k(i,j) = f_k y_k(i,j)$ for each $(i,j) \in \mathscr{A}$ and $k \in \mathscr{K}$. Based on Constraint (5), the upper bound of f_p is the maximum of the minimum arc capacities. Therefore, the upper bound of f_p will be $\Gamma = \max_{(i,j)\in\mathscr{A}} \min_{t\in\mathscr{T}} \mathscr{U}^t(i,j)$. Now using Proposition 1, Constraint (5) can be replaced by following constraints:

$$\sum_{k\in\mathscr{K}} w_k(i,j) \le \min_{t\in\mathscr{T}} \mathscr{U}^t(i,j), \qquad \forall (i,j) \in \mathscr{A}, \tag{12}$$

$$f_k + \Gamma(y_k(i,j) - 1) \le w_k(i,j) \le \Gamma y_k(i,j), \qquad \forall (i,j) \in \mathscr{A}, \forall k \in \mathscr{K}, \tag{13}$$

$$0 \le w_k(i,j) \le f_p, \qquad \forall (i,j) \in \mathscr{A}, \forall k \in \mathscr{K}, \tag{14}$$

5 Computational results

The numerical results are divided into two parts. First, it is described how the preprocessing algorithm works on a small network. In the second part, the performance of the proposed decision making tool is tested on a sample network with different scenarios for incidents. All the algorithms are implemented in a C++ environment and CPLEX 12.1 is used to solve the mathematical model. All experiments are made on a Windows Workstation with 2.83 GHz Intel Xeon Quad CPU and 16 GB RAM.

5.1 A numerical example of the preprocessing algorithm

Figure 3 is the sample network used to illustrate the preprocessing algorithm. It has three source nodes ($\mathscr{N}_s = \{1,2,3\}$), five intermediate nodes, and two destination nodes ($\mathscr{N}_d = \{9,10\}$). The amount of supplies in safe nodes 1, 2, and 3 are 350, 185, and 200, respectively. The capacity of both destination nodes 9 and 10 is 750.

Table 2 shows an evacuation plan for the sample network constructed by utilizing the evacuation route planning algorithm presented in Baharnemati and Lim (2011). The optimized schedule to evacuate all residents from the three source nodes includes three separate paths for the first node, two paths for the second node, and three paths for the third node. For node 1, residents are expected to leave the location taking any of the three paths at time 0. Both path $1-5-6-7-8-10$

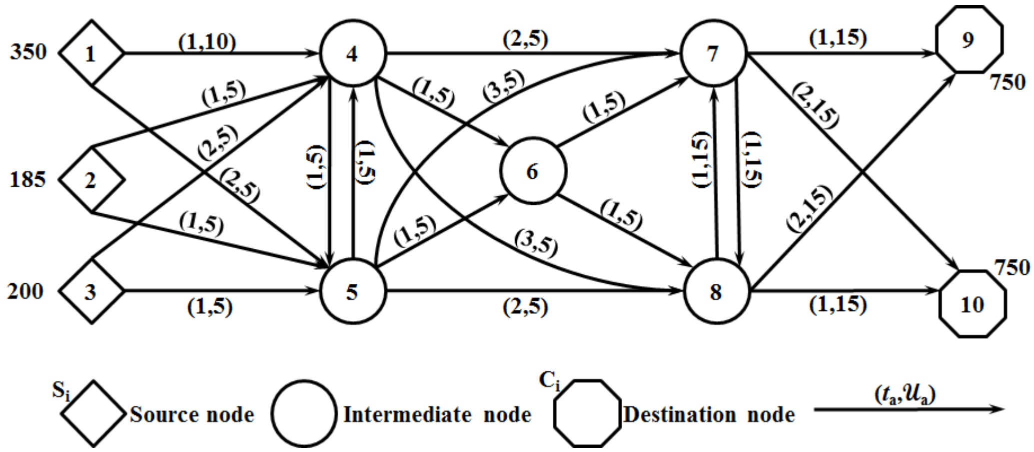

Figure 3: A sample network

and path $1-4-6-8-9$ allow five vehicles per unit time interval and path $1-4-5-8-10$ take four vehicles at each time interval. Based on this schedule, there will be 14 vehicles leaving node 1, 8 vehicles from node 2, and 8 vehicles from node 3 at each time interval for the duration of the scheduled time on the right column. Note that the expected travel time of each path can be different. Utilizing all paths as scheduled will ensure optimal evacuation for the given evacuation plan horizon, which is 30 units ($T = 30$) for this example. As mentioned before, the value of T is assumed to be given to this problem as an input parameter, which can be optimized using the bisection method (Baharnemati and Lim (2011)). Hence, schedule [0-23] of the first case (path $1-5-6-7-8-10$) means that there are 5 vehicles loaded per unit time for 24 times, which will result in 120 total vehicles evacuated from node 1 via this path.

Table 2: An evacuation plan for the sample network

Path	Flow rate (vehicles/unit_time)	Schedule
1-5-6-7-8-10	5	$0-23$
1-4-6-8-10	5	$0-25$
1-4-5-8-10	4	$0-24$
2-5-7-10	3	$0-23$
2-4-8-9	5	$0-23$
3-4-7-9	5	$0-24$
3-5-7-9	2	$0-24$
3-5-8-9	1	$0-24$

Suppose now that an accident was reported on arc $(7, 8)$, which blocks the use of the arc for time interval $[20, 27]$. Scanning through the list of the selected paths, path $1-5-6-7-8-10$ is affected by this incident which belongs to Case 6 in Table 1. Since travel time from node 1 to node 7 (the starting node of the arc) on this path is $4 = 2+1+1$ (i.e. $tr_1 = 4$), $[4, 27]$ is the time period that evacuees reach node 7. Therefore, evacuees who are scheduled to arrive at node 7 after time 20 will not be able to reach their destination (node 10). This motivates generating an alternative path for them to use during the time interval $[20, 27]$. Under this circumstance, Procedure 2 will be executed to update the evacuation static network and dynamic network. Then, a new commodity is created with node 7 being a new source node and node 10 being the destination node with new schedule $[20, 26]$ with supply of 35 $(= (\Theta_5^2 - \Theta_5^1 + 1) \cdot f_p(5))$. Next, the multi-commodity optimization model is constructed and solved based on the updated evacuation network and the commodity to generate an alternative path $(7-10)$ with the corresponding flow rate of 5 per unit time interval.

5.2 Numerical results for the proposed rerouting decision making tool

In this section, a sample network shown in Figure 4 is constructed based on the evacuation network of the Greater Houston area. This network has 42 nodes which are connected by 106 arcs. There are 13 source nodes and four safe nodes in the network. The total number of vehicles located at the source nodes are 5660. An evacuation plan is obtained by utilizing the evacuation route planning algorithm presented in (Baharnemati and Lim, 2011). The developed evacuation plan shown in Table 3 is based on a time horizon of 152 ($T = 152$) and a fixed flow rate for the evacuation paths.

To test the performance of the proposed decision making tool, random scenarios are generated. In each scenario, one arc is randomly selected with a randomly generated incident time period. Then, the decision making tool is utilized on the scenarios to find the alternative paths if they are required. The discussions about the random scenarios are as follow.

Scenario 1: impacted arc is $(29, 28)$ **with** $TP_1 = [147, 150]$ In this scenario, no action is required because there is one impacted arc that is not associated with any of the evacuation paths.

Scenario 2: impacted arc is $(25, 26)$ **with** $TP_1 = [36, 76]$ This case belongs to Case 6 of Table 1. After applying the preprocessing procedure, it is recognized that some of the evacuees on path $(10-30-31-32-25-26-24-41)$ are affected by the incident. A commodity is created

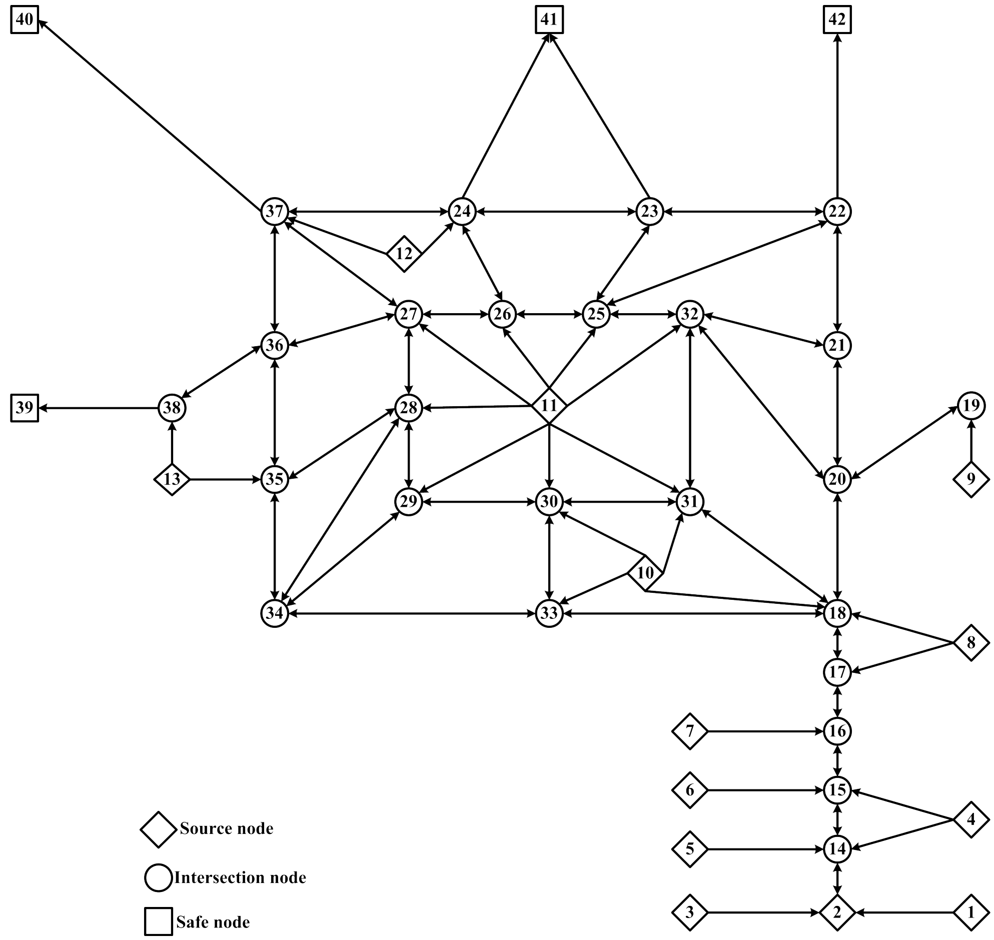

Figure 4: A sample network based on the evacuation network of Greater Houston area

Table 3: An evacuation plan for the sample network

No.	Supply	Path	Flow rate	Schedule
1	10	1-2-14-15-16-17-18-33-30-31-32-25-22-42	1	0 – 9
2	10	2-14-15-16-17-18-20-21-22-42	1	0 – 9
3	10	3-2-14-15-16-17-18-20-21-22-42	1	0 – 9
4	10	4-15-16-17-18-20-32-25-22-42	1	0 – 9
5	10	5-14-15-16-17-18-33-30-31-32-25-22-42	1	0 – 9
6	10	6-15-16-17-18-33-30-31-32-25-22-42	1	0 – 9
7	350	7-16-17-18-31-32-25-23-24-41	3	0 – 116
8	350	8-18-20-21-22-42	3	0 – 116
9	350	9-19-20-21-22-23-41	3	0 – 116
10	350	10-30-31-32-25-26-24-41	3	0 – 116
11	1400	11-25-23-41	5	0 – 136
		11-27-37-24-41	6	0 – 110
12	1400	12-37-40	10	0 – 139
13	1400	13-38-39	10	0 – 139

for the evacuees on this path by the following specifications: Source node (25), Destination node (41), Supply amount (123), and Schedule [36,76]. The multi-commodity network flow optimization model found an alternative path $(25-23-41)$ with flow 2 and schedule [36,76]. Using this new path, 82 vehicles can be rerouted. After time 76, arc $(25, 26)$ becomes usable again and evacuees can continue using the original path.

Scenario 3: impacted arc is $(21, 22)$ **with** $TP_1 = [68, 91]$ After applying the preprocessing procedure, it is recognized that the evacuation path $(2-14-15-16-17-18-20-21-22-42)$ may be affected by the incident. The travel time information reveals that the evacuation through this path has been completed before the incident was reported (Case 1 in Table 1). Furthermore, the evacuation paths of source nodes 3, 8, and 9 are affected by the incident. Since they belong to Case 6, three commodities are created by the following specifications:

Commodity 1: Source (21), Destination (42), Supply of 24, Schedule: [68,91]

Commodity 2: Source (21), Destination (42), Supply of 72, Schedule: [68,91]

Commodity 3: Source (21), Destination (41), Supply of 72, Schedule: [68,91]

The multi-commodity network flow optimization model found two alternative paths. The first alternative path is $(21-32-31-30-29-28-27-26-25-22-42)$ with a flow rate of 1 with schedule $[68, 91]$. The second alternative path is $(21-32-31-30-29-28-27-26-25-23-41)$ with a flow rate of 2 with schedule $[68, 91]$. As a result, 24 and 48 vehicles can travel via these paths to reach their destinations 42 and 41, respectively. After time 91, the evacuees continue following their original path towards their destinations because the network recovered to the normal state.

The occurrence of incidents in transportation networks during a massive evacuation is unavoidable. Therefore, it is important to have a decision making tool to quickly prepare an alternative evacuation plan when an incident occurs. The computational results showed that the proposed decision making tool has the capability to meet such a need. The alternative paths reroute the stranded evacuees on the affected paths to their destination without getting stuck in gridlock on the evacuation path.

6 Summary

An evacuation reroute planning approach has been presented to be used in the event of an incident causing delays within the evacuation network. A decision making tool was proposed composed of two elements: a network preprocessing algorithm and a network flow optimization model. A preprocessing algorithm with two independent procedures was developed to update the evacuation network in terms of residual capacities of nodes and arcs and changes at the source nodes and destination nodes. The network flow optimization model was developed to find a set of alternative paths and their corresponding flow rates. Since the optimization model of the problem was MINLP, a linear reformulation technique was used to linearize the original model to reduce the computational burden. Numerical experiments were conducted using several test instances. This approach was further tested on the actual evacuation network of the Greater Houston area. Various incident scenarios were generated and the results were discussed to show that the proposed tool can effectively generate alternative paths to avoid getting stuck in gridlock on the evacuation path.

References

Akgün, V., Parekh, A., Batta, R. and Rump, C. M. (2007), 'Routing of a hazmat truck in the presence of weather systems', *Computers & operations research* **34**(5), 1351–1373.

Baharnemati, M. and Lim, G. (2011), Hurricane Evacuation Planning: A Network Flow Optimization Approach, *in* 'Proceedings of the 61st IIE Annual Conference', Institute of Industrial Engineering.

Cova, T. and Johnson, J. (2003), 'A network flow model for lane-based evacuation routing', *Transportation research part A: Policy and Practice* **37**(7), 579–604.

Desai, S. and Lim, G. J. (2013*a*), 'Solution time reduction techniques of a stochastic dynamic programming approach for hazardous material route selection problem', *Computers & Industrial Engineering* **65**(4), 634–645.

Desai, S. S. and Lim, G. J. (2013*b*), 'An information based routing model for hazardous material route selection problem', *Industrial and Systems Engineering Review* **1**(1), 1–12.

Galindo, G. and Batta, R. (2013), 'Review of recent developments in or/ms research in disaster operations management', *European Journal of Operational Research* **230**(2), 201–211.

Glover, F. (1975), 'Improved linear integer programming formulations of nonlinear integer problems', *Management Science* pp. 455–460.

Kok, A., Hans, E. and Schutten, J. (2012), 'Vehicle routing under time-dependent travel times: the impact of congestion avoidance', *Computers & operations research* **39**(5), 910–918.

Lim, G. J., Rungta, M. and Baharnemati, M. R. (2015), 'Reliability analysis of evacuation routes under capacity uncertainty of road links', *IIE Transactions* **47**(1), 50–63.

Lim, G., Zangeneh, S., Baharnemati, M. and Assavapokee, T. (2012), 'A network flow optimization approach for a short notice evacuation planning', *European Journal of Operational Research* **223**(1), 234–245.

Ng, M. and Waller, S. (2010), 'Reliable evacuation planning via demand inflation and supply deflation', *Transportation Research Part E: Logistics and Transportation Review* **46**(6), 1086–1094.

Rungta, M., Lim, G. and Baharnemati, M. (2012), 'Optimal egress time calculation and path generation for large evacuation networks', *Annals of Operations Research* **201**(1), 403–421.

Yao, T., Mandala, S. and Chung, B. (2009), 'Evacuation transportation planning under uncertainty: a robust optimization approach', *Networks and Spatial Economics* **9**(2), 171–189.

Zheng, H. and Chiu, Y.-C. (2011), 'A network flow algorithm for the cell-based single-destination system optimal dynamic traffic assignment problem', *Transportation Science* **45**(1), 121–137.