A Pool Strategy of Microgrid in Power Distribution Electricity Market

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Abstract—This paper discusses a market-based pool strategy for a microgrid (MG) to optimally trade electric power in the distribution electricity market (DEM). The increasing penetration levels of distributed energy resources (DERs) and MGs in distribution system (DS) stress distribution system operator (DSO) and require higher levels of coordinated control strategies. The distribution system operator has limited visibility and control over such distributed resources. To reduce the complexity of the system and improve the efficiency of the electricity market operation, we propose a decentralized pool strategy for an MG to integrate with a distribution system through a market mechanism. A market-based interaction procedure between MGs and DS is developed for MGs as price-makers to find an optimal bidding/offering strategy efficiently. To achieve a market equilibrium among all entities, we initially cast this problem as a bi-level programming problem, in which the upper level is an MG optimal scheduling problem and the lower level presents a DEM clearing mechanism. The proposed bi-level model is converted to a single mixed-integer model which is easier to solve. Uncertainties associated with MG’s rivals’ offers and demands’ bids are considered in this problem. The solution results from a modified IEEE 33-Bus distribution system are presented and discussed. Finally, some conclusions are drawn and examined.

Keywords—bidding and offering strategies, bi-level programming, distribution electricity market, distribution locational marginal prices, mathematical programs with equilibrium constraints, Microgrid, Stackelberg game.

NOMENCLATURE

Indices

- $b$ The node subscript index in DS, $b \in B$
- $b'$ The node location where $j$-th microgrid is located
- $j$ MG subscript index connected with DS, $j \in J$
- $k$ Generation unit subscript index in MG $j$-th, $k \in K_j$
- $l$ Consumer subscript index in DS $l \in L$
- $m$ Utility node subscript index connected with DS $m \in M$
- $n$ Distributed generator (DG) subscript index in DS, $n \in N$
- $t$ Index for time periods $t \in T$

Parameters

- $D^p_{jt} / D^q_{jt}$ Real/reactive power consumption for $j$-th MG in time $t$
- $K_{ij}$ Incidence matrix
- $P^\text{min}_{(j)} / P^\text{max}_{(j)}$ Maximum/minimum real power output in time $t$
- $Q^\text{min}_{(j)} / Q^\text{max}_{(j)}$ Maximum/minimum reactive power outputs in time $t$
- $UR_{(j)} / DR_{(j)}$ Generator ramp up/down rates
- $\delta_{(j)}^G$ Marginal cost of generation unit $i$ in MG in time $t$
- $\delta_{(j)}^L$ Marginal profit of consumer $l$ in DS in time $t$
- $\delta_{(j)}^M$ Marginal cost of DG $n$ in DS in time $t$
- $\delta_{(j)}^O$ Marginal cost of utility $m$ in time $t$
- $\alpha_{(j)}$ Large positive constant

Sets

- $B$ DS node set $B = \{1, \ldots, NB\}$, $NB$ is number of node; ($B^+)$ is subset of $B$, means node with ($)$ component.
- $J$ MG set $J = \{1, \ldots, NM\}$, $NM$ is number of MG.
- $K_j$ Generation unit set $K_j = \{1, \ldots, NDG_j\}$ in $j$-th MG, $NDG_j$ is number of generation units
- $L$ Consumer set in DS $L = \{1, \ldots, NL\}$, $NL$ is number of load in DS
- $M$ Utility set in DS $M = \{1\}$
- $N$ DG in DS set $N = \{1, \ldots, ND\}$, $ND$ is number of distributed generation units in DS
- $T$ Time period set $T = \{1, \ldots, NT\}$, $NT$ is number of time

Variables

- $I_{ki}$ Binary variable associated with generator $k$ state
- $P_{li}$ Real power flow at node $b$ in time $t$
- $P_{li} / Q_{li}$ Real/reactive power output of the MG, DG, utility, consumer in time $t$
- $P_{li} / Q_{li}$ Real/reactive power injection at node $b$ in time $t$
- $Q_{li}$ Reactive power flow at node $b$ in time $t$
- $V_{bi}$ Voltage magnitude at node $b$ in time $t$
- $\alpha_{(j)}$ Offering/Bidding price MG $j$ submitted
βₜ

The j-th MG marginal operation cost in time t

λᵢₜ / 2

Lagrangian multipliers associated with real/reactive power balance constraints

μₗₜ, μₗₜ max, μₗₜ min

Lagrangian multipliers associated with real(p)/reactive(q) power output of the MG, DG, utility, consumer constraints

πₗₜ, πₗₜ min, πₗₜ max

Lagrangian multipliers associated voltage magnitude constraints

ρₗₜ

Auxiliary binary variables to linearize complementary slackness constraints

τ(ᵢₜ)

Auxiliary binary variables to linearize complementary slackness constraints

ω(ᵢₜ)

Auxiliary binary variable of MG power exchange cost with distribution system

Ω / Q

I. INTRODUCTION

The microgrid concept is proposed to facilitate the integration of distributed energy resources into the electricity grid, which can reduce transmission grid losses and overcome limitations in distribution system [1]. By integrating distributed energy resources into microgrids with smart central controllers and smart sensors, MGs can provide highly reliable electrifications which can guide customers to lower their operation costs and utilize electricity more efficiently [2]. MGs can also benefit power system through profitable and environmentally friendly services [3], higher power system resiliency [4], less transmission and distribution costs [5], fewer carbon emissions by the use of renewable power resources [6], and utilization of electrification in rural areas [3]. With all of these benefits, microgrids can be expected to be used in a wide variety of electrical environments [7].

Microgrid can work in either Islanding or Grid-connected mode at the point of common coupling (PCC) [8]. To ensure a secure MG operation in a centralized manner [9], MG has three control levels: primary, secondary and tertiary. The primary and secondary controls are able to maintain the frequency/voltage of the MG. As the primary focus of this paper, two goals of tertiary control are (1) to optimally manage the power flow between the MG and the utility grid [10], and (2) to minimize microgrid operation cost while providing high-quality service to various types of customers in uncertain environments. Although the benefits of optimally scheduling MGs have been reported in the literature [1], [4], [9], [11], and [12], drawbacks of the existing approaches are that they are limited to MG scheduling, and do not address the interactions between microgrids and distribution system concerning power coordinated operation strategy and distribution electricity market price policy. With regard to power coordinated operation strategy, the distribution system was assumed as an infinite bus that can provide unlimited power supply/load to mitigate any power imbalance in MGs [10]. However, this assumption has a crucial flaw because the distribution system operator, in fact, has the physical capacity limitation to do so. Furthermore, the distribution system operator does not have an incentive to provide power beyond the economically optimal level. As for the distribution electricity market pricing policy, due to the presence of price uncertainty and its consequences, the market price between microgrid and the distribution system is not known in advance. Consequently, the current practice of bidding/offering pricing strategies may not be optimal.

The coordinated strategy can be economically beneficial to both microgrids and the distribution system [13]. Such benefits of using a decentralized coordinated management (DCM) include higher profits [14], improved efficiency of DERs and reduced complexity of distribution network operation [15], and improved system reliability [16]. The current literature on DCM assumes fixed pricing strategy. However, the fixed pricing approach does not guarantee optimality because it is difficult to include the abnormal conditions such as overloading, islanding, component outages as well as load uncertainty and volatility of non-dispatchable generation units. These conditions can provide market power or non-beneficial outcomes for decentralized coordinated management participants. Hence, there is a clear need for an approach that considers both the coordinated management strategy and the distribution electricity market pricing policy.

A successful distributed electricity market requires a good pricing policy. Overall pricing schemes in the existing industrial distributed electricity markets can be found in [17]. Furthermore, a study has been reported to compare different distributed electricity market designs and pricing policies [18]. The pricing policies can be categorized as price-based and market-based management. The price-based management is an efficient way to handle the DEM by using fixed forecast price [9]-[12]. However, this approach is not well suited when the microgrid penetration in the distribution network is high. Therefore, the market-based management was proposed as an alternative [19]. The market-based DEM with dynamic pricing is more flexible than the price-based DEM. However, the proposed market-based bidding strategy for MG does not guarantee optimality because the power interaction between MG and distribution system is determined by distribution system only. Furthermore, there is no explicit optimal bidding curve creation strategy which has the significant impact on distributed electricity market operation. Another bidding strategy for microgrid as price-taker in market-based wholesale market can be found in [20]. Nonetheless, the MG is not widely accepted by high voltage wholesale market directly because: 1) MG’s capacity is limited [21] and 2) the high voltage network is not designed for bi-directional power flow. The distribution system fits microgrid and other DERs with advanced distributed system operator and the distribution market operator (DMO), which is helpful in managing price information among market participants. In reality, the MGs and other DERs are two primary competing power suppliers in DEM, which constitute an oligopolistic distribution electricity market, leading to imperfect competition. An imperfect competitor is in fact a price-maker [22] whose offering/bidding strategy has the ability to influence the market profile defined by aggregated behaviors of all market participants. Therefore, a new market-based mechanism is needed so that the MGs can impact the DEM’s market price. This paper attempts to shed light on a realistic economical behavior of an MG in the distributed electricity market beyond.
the proposed market-based scheme [19]. Because an MG is a prosumer in the DS, a combined offer-and-bid pair can be submitted to the distributed electricity market. This necessitates a new strategy, in which an MG plays as a price-maker in the market-based distribution electricity market.

This problem can be cast as the Stackelberg game where a microgrid plays the role of a leader, while competitors and consumers are the followers [23]. Under this framework, bi-level programming is used to formulate the optimal offering strategy problem [24]. A bi-level programming model can be converted into mathematical programming with equilibrium constraints (MPEC) [25], which is a highly non-convex optimization problem [26]. To reduce computational burden for solving the MPEC model, a binary expansion solution approach proposed by [27] can be used to convert the MPEC model into a mix-integer programming (MIP) model, which gives a global optimal solution.

Therefore, this paper proposes a new coordinated pool strategy, in which a microgrid plays as a price-maker in the market-based distributed electricity market. Considering MGs as strategic prosumers, a MIP model is developed to maximize the benefits for MGs from trading power in DEM through an optimal bidding/offering strategy. A modified bidding/offering policy is provided to overcome drawbacks of existing strategies.

The remainder of this paper is organized as follows. Section II presents the model outline and assumptions. Section III formulates the bi-level programming problem and solving algorithm. The model is tested under price uncertainty as well as MG contingencies such as islanding in Section IV. Relevant conclusions are discussed in Section V.

II. OUTLINE AND ASSUMPTIONS

The decentralized pool strategy we propose in this paper has two levels as seen in Fig.1: a microgrid level and a distribution system level. In the MG level, the microgrid operator (MGO) is in charge of optimally scheduling MG-owned DGs and local consumers. In the DS level, the distribution system operator takes care of interactions between the DS and its participants. The distribution network operator (DNO) is responsible for power flow, and the distribution market operator is responsible for market regulation. The DMO enables competitive access to markets and the optimal use of DERs on distribution networks. Under the distribution system operator (DSO) model, the operator accepts a wide range of management rules beyond the network operation of DNO and market responsibility of DMO. The DSO can help provide reliable and secure operations to the DS by enabling highly reliable networks, flexible DERs and demand response program under a competitive market environment.

The pool bidding [28] and the coordinated management [14] are used together to solve the problem of high penetration levels of MGs in DS. The MGs are strategic players whose bids/offers are subject to market profile, which is decided by nonstrategic players such as the DS customers, DS-owned DGs, and high voltage utility nonstrategic players. The distribution electricity market uses a price signal such as distribution locational marginal price (DLMP) as feedback to MG’s bids/offers. The DLMPs are widely used as price signals among market participants or between the market operator and the market agent [29].

The microgrid as a price-maker with an independent operator has autonomy to make its own scheduling and bidding/offering decisions in response to distribution system operation states and market price signals which leverage the MGs’ transactive capabilities in the distributed electricity market [29]. As a result, it can help the distribution system operator reduce the decision burden and network complexity. At the same time, the power pool regulation at the distribution system level defines standards for processing and evaluating electricity price bids [30], which ensure the microgrids and distribution generators can freely participate in the distributed electricity market.

The key components to implement these regulations are DSO, DNO and DMO. The state of art distribution system operators can perform active managements including market regulations and demand response with greater flexibility and capability between supply and demand [31]. Such examples include Distributed System Platform Provider proceeding proposed by the New York Public Service Commission [32], the Multi-Microgrid in Chicago including the IIT Campus Microgrid (ICM) and the Bronzeville Community Microgrid (BCM) [33], and European Distribution System Operators advocated by the European Union [31]. Some distribution network operator’s responsibilities like power balancing and network operation can also be taken by the DSO. It is too early to conclude that the DNO will be entirely replaced by the DSO [34] as the DNO’s contributions in security and quality of supply and power flow management are significant [35]. In some distribution electricity markets such as Cornwall Local Energy Market [36] and TDI 2 [37], the DNO is successfully acting as the DSO to manage the distribution system. In our proposed framework, we adopted the concept of the transactive energy systems [29], which both DNO and DMO entities are defined under the unified DSO. The advanced DSO expands the conventional operational domain of the DNO and the DMO to enable a sound distribution system operation with high penetration levels of DERs. It also facilitates the MGs as prosumers to implement transactive exchanges.

![Fig.1. Decentralized pool strategy for distributed electricity market](image-url)
Compared with previous DEM management strategies, our proposed strategy has the following advantages:

- Having MG as a strategic player enables a bi-directional power flow between MG and DS, which can smooth out or shift the peak hour load.
- The MGs’ bidding/offering price based on DLMP reflects the exact market mechanism of the distribution electricity market. This approach helps the microgrid operator reduce its burden to determine the true market value of its power resources in trading in DEM.
- MGs as price-makers have direct influence on DEM price. However, influencing the price may create associated market risk due to price uncertainty, which MGs must take if it occurs.
- Efficiency of clearing the market can be improved by allowing competition among all the power source owners [19]. This can be done because the DMO can evaluate all the bids and offers ranging from the cheapest to the most expensive before transactions occur among all market participants.
- The separation of roles between a distribution network operator (technical functions: e.g. power flow) and a distribution market operator (market regulation) prevents the producers from abusing the market.

In this paper, the distribution system network is modeled with AC Distribution load flow [38]. The DS-owned DGs and MGs are primary power suppliers of distribution electricity market. Two main consumers are MG community load and DS spot load. The DEM pool is cleared hourly, day-ahead within the DistFlow framework. The hourly DLMPs reflect adequately distributed MGs’ influence to DEM. The 24-hour DLMPs are obtained through dual variables associated with real power balance constraints. The MG scheduling model includes most of its features, i.e., unit linearized operation cost, generator capacity limits, and generator ramping up/down rates. The paper assumes that DS-owned DGs offer with their marginal costs, and spot loads bid with forecast market prices. The MGs’ bids/offers are based on actual DLMPs of DEM. We use linearized operation cost offering curves for all generators and linearized bidding curves for all customers.

### III. Model and Solution Methodology

#### A. Bi-level Programming Model

The optimal bidding problem is formulated as a bi-level programming model as follows:

**ULPM:**

\[
\min \sum_{j} \sum_{t} \left( \sum_{i} D_{ij} - P_{ij} + \lambda_{ij}^{D} \cdot P_{ij} + VOLL \cdot D_{ij} \right) 
\]

\[
P_{ij} - P_{ij}^{min} \leq P_{ij} \leq P_{ij}^{max}, \forall t, k \in K, j \in J \]  

\[
Q_{ij}^{min} \leq Q_{ij} \leq Q_{ij}^{max}, \forall t, k \in K, j \in J \]  

\[
P_{ij} - P_{ij}^{min} \leq RU_{ij}, \forall t, k \in K, j \in J \]  

\[
P_{ij} - P_{ij}^{max} \leq RD_{ij}, \forall t, k \in K, j \in J \]  

\[
\sum_{k} P_{ij} \leq D_{ij}^{P} + P_{ij}, \forall t, k \in K, j \in J \]  

**LLPM:**

\[
\min \left( \sum_{j} \sum_{t} \left( \sum_{i} D_{ij} - P_{ij} + \lambda_{ij}^{D} \cdot P_{ij} + VOLL \cdot D_{ij} \right) \right) 
\]

\[
P_{ij}^{min} \leq P_{ij} \leq P_{ij}^{max}, \forall t, i \in J, j \in J \]  

\[
Q_{ij}^{min} \leq Q_{ij} \leq Q_{ij}^{max}, \forall t, i \in J, j \in J \]  

\[
P_{ij} - K_{ij} Q_{ij}^{min} \leq P_{ij} - K_{ij} Q_{ij}^{max}, \forall t, i \in J, j \in J \]  

\[
P_{ij} - P_{ij}^{min} \leq VOLL_{ij}^{D} - P_{ij}^{max}, \forall t, i \in J, j \in J \]  

\[
V_{ij}^{min} \leq V_{ij} \leq V_{ij}^{max}, \forall t, i \in J, j \in J \]  

The objective function of the upper level programming model (ULPM) is to minimize power generation cost of microgrids, power exchange cost at point of common coupling and load shedding cost. The power exchange cost is negative when MGs are extracting power from the distribution system or positive when MGs are exporting power to the DS. Dispatchable generators in MG are subject to real power output capacity constraint (2), reactive power output constraint (3), ramp up rate (4) and ramp down rate (5). Real power balance equations (6) together ensure that the power generated by DGs is used to supply the entire load and the power exchange at PCC. The DLMPs (\(\lambda_{ij}^{D}\)) are endogenously generated from the lower-level programming model (8) - (16) (LLPM), and the MG uses DLMPs as the base bidding/offering price (\(\lambda_{ij}^{D}\)). The real power and reactive power exchange at PCC belong to the feasible set defined by the LLPM as in constraint (8).

The LLPM presents the distribution system market clearing problem with the objective to maximize the social welfare (8), which consists of four terms. The first three terms represent the total cost for the DS: operation cost from DS-owned DGs, power exchange with MG, and the cost of extracting power from utility power system. The last item is total benefits obtained by supplying power to customers. Constraints (9) and (10) guarantee that the DGs’ outputs, MGs power exchange, utility extraction, and load requirement are within a capacity range. The constraints (11) - (16) are DistFlow equations that can be used to describe the complex power flows at each node for DS. Constraints (11) and (12) are real power injection and reactive power injection at each node. The possible equations to use are power balance equations, which can be written for real and reactive power for each bus. Constraints (13) and (14) are real and reactive power balance equations at each node, which guarantee the power balance. Constraint (15) is the node voltage equation. Voltage limits are defined in constraint (16). The justification of the linearized method for DistFlow can be found in [14].

Dual variables associated with each constraint are labeled next to the corresponding constraints: \(\mu_{ij}^{min}, \mu_{ij}^{max}, \mu_{ij}^{min}\).


\[ \mu_{t_i}^{\text{min}} \leq \lambda_{t_i}^{P} \leq \mu_{t_i}^{\text{max}} , \lambda_{t_i}^{P} \geq \lambda_{t_i}^{Q} \geq \lambda_{t_i}^{P} \geq \mu_{t_i}^{\text{min}} \text{ and } \pi_{t_i}^{\text{min}} \geq 0 . \]

It is noted that the LLPM is a linear programming model if the microgrids’ bidding/offering price \( \alpha_{t_i} \) is treated as input parameters. Thus, the LLPM can be replaced with Karush-Kuhn-Tucker (KKT) optimality conditions to formulate as an MPEC.

\section*{B. MPEC}

The KKT optimality conditions for LLPM are constructed as follows:

\[ \begin{align*}
\delta_{t_i}^{P} - \mu_{t_i}^{\text{min}} + \lambda_{t_i}^{P} - \lambda_{t_i}^{Q} &= 0 \quad \forall b \in B^+, \forall t \quad (17) \\
\delta_{t_i}^{Q} - \mu_{t_i}^{\text{min}} + \mu_{t_i}^{\text{max}} - \lambda_{t_i}^{Q} &= 0 \quad \forall b \in B^+, \forall t \quad (18) \\
\alpha_{t_i} - \mu_{t_i}^{\text{min}} + \lambda_{t_i}^{P} - \lambda_{t_i}^{Q} &= 0 \quad \forall b \in B^+, \forall t \quad (19) \\
-\delta_{t_i}^{P} + \mu_{t_i}^{\text{min}} + \lambda_{t_i}^{P} + \lambda_{t_i}^{Q} &= 0 \quad \forall b \in B^-, \forall t \quad (20) \\
\forall i \in J, L, M, N, \forall b \in J, L, M, N, \forall t \\
\lambda_{t_i}^{P} - \lambda_{t_i}^{P(b+1)} - r_{t_i} \pi_{t_i}^{V_i} &= 0 \quad \forall b \in J, \forall t \quad (22) \\
\lambda_{t_i}^{Q} - \lambda_{t_i}^{Q(b+1)} - x_{t_i} \pi_{t_i}^{V_i} &= 0 \quad \forall b \in J, \forall t \quad (23) \\
\pi_{t_i} - E_{t_i}^{(b+1)} - V_i^{\text{max}} - V_i^{\text{max}} &= 0 \quad \forall b \in J, \forall t \quad (24) \\
0 \leq \mu_{t_i}^{\text{min}} \perp (P_b - P_i^{\text{min}}) & \geq 0, \forall i \in J, L, M, N \quad (25) \\
0 \leq \mu_{t_i}^{\text{max}} \perp (P_b - P_i^{\text{max}}) & \geq 0, \forall i \in J, L, M, N \quad (26) \\
0 \leq \mu_{t_i}^{\text{min}} \perp (Q_b - Q_i^{\text{min}}) & \geq 0, \forall i \in J, L, M, N \quad (27) \\
0 \leq \mu_{t_i}^{\text{max}} \perp (Q_b - Q_i^{\text{max}}) & \geq 0, \forall i \in J, L, M, N \quad (28) \\
0 \leq \pi_{t_i}^{\text{min}} \perp (V_i - V_i^{\text{min}}) & \geq 0, \forall b \in J, \forall t \quad (29) \\
0 \leq \pi_{t_i}^{\text{max}} \perp (V_i - V_i^{\text{max}}) & \geq 0, \forall b \in J, \forall t \quad (30) \\
(9) - (16) \\
\mu_{t_i}^{\text{min}}, \mu_{t_i}^{\text{max}}, \lambda_{t_i}^{P}, \lambda_{t_i}^{Q}, \pi_{t_i}^{\text{min}}, \pi_{t_i}^{\text{max}}, \omega_{t_i}^{\text{max}} \geq 0 \quad (32)
\end{align*} \]

The KKT optimality conditions contain stationarity (17)-(24), complementary slackness (25)-(30), primal feasibility (31), and dual feasibility (32). The bi-level programming model is replaced with (1) - (9) and (17) - (32) as MPEC. The MPEC is a non-convex problem, thus the linearize technics are needed to solve the problem.

\section*{C. Linear Reformulation of MPEC}

The nonlinearity of MPEC comes from two parts: MGs’ bidding/offering in upper-level objective function \( \lambda_{t_i}^{P} P_{t} \), and complementary slackness part in lower-level KKT equivalent constraints (25) - (30).

To linearize \( \lambda_{t_i}^{P} P_{t} \), we applied strong duality method using \( \delta_{t_i}^{P} P_{t} \), and dual feasibility (32). The bi-level programming problem which can be solved by using some commercial software packages. The MIP formulation is as follows:

\[ \begin{align*}
\min & \sum_{i} \delta_{t_i}^{P} P_{t} + \sum_{i} \sum P O L L D + \mu_{t_i}^{\text{min}} + \lambda_{t_i}^{P} + \pi_{t_i}^{\text{min}} + \pi_{t_i}^{\text{max}} + \omega_{t_i}^{\text{max}} \geq 0 \quad (32)
\end{align*} \]

Subject to: (2) - (7), (17) - (24), (31) - (32), (34) - (45)

\section*{D. MG Bidding/Offering Strategy}

The microgrid is prosumer such that (1) it can submit offers to the distribution market operator when it exports power to the DS or (2) it can submit bids to the distribution market operator when it extracts power from the DS. The bidding/offering prices for MGs in bi-level model always coincide with DLMPs. However, this bidding/offering strategy may result in a solution that is not practical for the following reasons: (i) a flat offer curve may result in multiple solutions; (ii) some incentive(s) or even protective policy are necessary to maintain the profitability of MGs; (iii) no way to ensure the market clearing to have increasing offer curves or decreasing bid curves; and (iv) bidding/offering curves in practice are more complicated than the linearized or piecewise linearized curve adopted in our bi-level model.
To provide a remedy to the issues, we propose a direct and simple bidding/offering strategy for microgrids to find bidding/offering price (\(\alpha'_{jt}\)) based on two pieces of price information: MG corresponding marginal cost (\(\beta_{jt}\)) and DLMPs (\(\lambda_{jt}\)) in DS. The marginal cost of an MG can be obtained at the intersection of the aggregated marginal cost curve of its DGs and the maximum capacity of its PCC. DLMPs are declared at the DS level through the DEM clearing mechanism. Hence, a modified bidding/offering strategy for MGs is proposed as follows:

**Offering Strategy:**

1. If \(P_j = 0\), it indicates that either MG \(j\)-th is on islanding mode or bidding/offering prices are not accepted. If \(\beta_{jt} < \lambda_{jt}\), then \(\alpha'_{jt} = \lambda_{jt}\). If \(\beta_{jt} > \lambda_{jt}\), then \(\alpha'_{jt} = \beta_{jt}\). This keeps MG \(j\)-th from being accepted at a higher price.

2. If \(0 < P_j \leq P_{j_{\text{max}}}^{\text{DLMP}}\) and \(\beta_{jt} < \lambda_{jt}\), it indicates that MG \(j\)-th is working on grid connected mode. The MG \(j\)-th is transferring power to DS where the market price is relatively higher. Then we set \(\alpha'_{jt} = \lambda_{jt} + \varepsilon\) to make sure DS is willing to take more.

3. If \(0 < P_j \leq P_{j_{\text{max}}}^{\text{DLMP}}\) and \(\beta_{jt} > \lambda_{jt}\), it indicates that MG \(j\)-th is generating power with higher cost to supply DS loads at a lower price. Then we set \(\alpha'_{jt} = \beta_{jt}\) to maintain MG’s profitability in the market.

**Bidding Strategy:**

4. If \(P_{j_{\text{min}}}^{\text{DLMP}} < P_j < 0\) and \(\beta_{jt} < \lambda_{jt}\), it indicates that the MG \(j\)-th is extracting power from DS with higher cost even though it has a cheap power source available inside. Then we set \(\alpha'_{jt} = \beta_{jt}\) to maintain the profitability of MG \(j\)-th.

5. If \(P_{j_{\text{min}}}^{\text{DLMP}} < P_j < 0\) and \(\beta_{jt} > \lambda_{jt}\), it indicates that MG \(j\)-th is extracting power from DS rather than generating power itself with higher cost. Then MG will bid with price \(\alpha'_{jt} = \lambda_{jt} + \varepsilon\). The decreasing bid can encourage DS to export more power to MG \(j\)-th.

It is noted that \(\varepsilon\) is a very small positive constant, e.g., \(10^{-5}\).

**E. Uncertainty Modeling**

When MGs participate in DEM as price makers, uncertainties associated with their rivals (DS-owned DGs) and customers in DS highly affect the bidding/offering decisions that MGs make. The bidding/offering prices made by rivals and customers may fluctuate with load consumption changes. The probability distribution of a real-time market price is not precisely known and may vary with unpredictable system conditions in short term operation such as network, load and units availabilities [20]. Hence, a robust optimization method is more appropriate to handle these uncertainties. The offering price of DGs can be modeled as a summation of two terms \(\delta_{jt}^{\text{o}} + \delta_{jt}^{\text{u}}\xi_{jt}\), where \(\delta_{jt}^{\text{o}}\) is a predicted offering price, \(\xi_{jt}\) is an unknown variable associated with price uncertainty, and \(\delta_{jt}^{\text{u}}\) is a scale parameter. In setting up a robust optimization model, the uncertainty set for \(\xi_{jt}\) is modeled as follows:

\[
U_{jt} = \{\xi_{jt} : \hat{\xi}_{jt} \in [-d_{jt}, d_{jt}]\}, \quad (47)
\]

Above, parameter \(d_{jt}\) controls the level of uncertainty. If \(d_{jt} = 0\), the price uncertainty is ignored. If \(d_{jt} = 1\), it means that all price uncertainties are taken into account. Similarly, the customers’ offering price (\(\hat{\delta}_{jt}^{\text{c}}\)) can be modeled as \(\delta_{jt}^{\text{c}} + \delta_{jt}^{\text{c}}\xi_{jt}\). The uncertainty set for \(\hat{\xi}_{jt}\) is defined as

\[
U_{jt} = \{\hat{\xi}_{jt} : \hat{\xi}_{jt} \in [-d_{jt}, d_{jt}]\}. \quad (48)
\]

Consequently, the objective function that minimizes the worst-case scenario [39] can be stated as:

\[
\min \sum_{jt} \alpha_{jt} P_{j_{\text{max}}}^{\text{DLMP}} + \sum_{jt} \delta_{jt}^{\text{c}} P_{jt} + \max_{\xi_{jt}, \hat{\xi}_{jt} \in U_{jt}} \left( \sum_{jt} (\delta_{jt}^{\text{c}} + \delta_{jt}^{\text{c}}\xi_{jt}) P_{jt} - \sum_{jt} (\delta_{jt}^{\text{c}} + \delta_{jt}^{\text{c}}\hat{\xi}_{jt}) P_{jt} \right) \quad (49)
\]

**Proposition:** In objective function (49),

\[
\max_{\xi_{jt}, \hat{\xi}_{jt} \in U_{jt}} \left( \sum_{jt} (\delta_{jt}^{\text{c}} + \delta_{jt}^{\text{c}}\xi_{jt}) P_{jt} - \sum_{jt} (\delta_{jt}^{\text{c}} + \delta_{jt}^{\text{c}}\hat{\xi}_{jt}) P_{jt} \right)
\]

is equivalent to \(\sum_{jt} (\delta_{jt}^{\text{c}} + d_{jt}\hat{\xi}_{jt}) P_{jt} - \sum_{jt} (\delta_{jt}^{\text{c}} - d_{jt}\hat{\xi}_{jt}) P_{jt}\).

**Proof:** See Appendix.

Therefore, the robust optimization model for the LLMP is:

\[
\min \sum_{jt} \alpha_{jt} P_{j_{\text{max}}}^{\text{DLMP}} + \sum_{jt} \delta_{jt}^{\text{c}} P_{jt} + \sum_{jt} (\delta_{jt}^{\text{c}} + d_{jt}\hat{\xi}_{jt}) P_{jt} - \sum_{jt} (\delta_{jt}^{\text{c}} - d_{jt}\hat{\xi}_{jt}) P_{jt}\]

Subject to (10) - (17)

The objective function states that DS-owned DGs attempt to maximize their profits by offering the highest price possible. In the meantime, the customers wish to decrease its bidding price to lower the energy cost. Following the linearization process discussed in Section III (B&C), the robust MIP model (53) is essentially the same as (47) by replacing \(\delta_{jt}^{\text{c}}\) with \(\delta_{jt}^{\text{c}} + d_{jt}\hat{\xi}_{jt}\) and \(\delta_{jt}^{\text{c}}\) with \(\delta_{jt}^{\text{c}} - d_{jt}\hat{\xi}_{jt}\) formulated as follows:

\[
\min \sum_{jt} \delta_{jt}^{\text{c}} P_{jt} + \sum_{jt} \sum_{vt} \sum_{jt} \lambda_{jt} \sum_{jt} \Omega' \quad (52)
\]

\[
\text{s.t. (2) - (7), (18), (19), (21) - (24), (31) - (32), (34) - (45)}
\]

\[
\Omega' = \Omega(\delta_{jt}^{\text{c}} \rightarrow \delta_{jt}^{\text{c}} + d_{jt}\hat{\xi}_{jt}, \delta_{jt}^{\text{c}} \rightarrow \delta_{jt}^{\text{c}} - d_{jt}\hat{\xi}_{jt})
\]

\[
\delta_{jt}^{\text{o}} + d_{jt}\hat{\xi}_{jt} - \mu_{jt}^{\text{min}} + \mu_{jt}^{\text{max}} - \lambda_{jt}^{\text{c}} = 0 \quad \forall b \in B^{*}, \forall t \quad (53)
\]

\[
-\delta_{jt}^{\text{c}} + d_{jt}\hat{\xi}_{jt} - \mu_{jt}^{\text{min}} + \mu_{jt}^{\text{max}} + \lambda_{jt}^{\text{c}} = 0 \quad \forall b \in B^{*}, \forall t \quad (54)
\]

**IV. NUMERICAL EXPERIMENTS**

The model is tested on a modified IEEE 33-bus distribution system with three microgrids and five DGs in the system [40]. The model was solved using IBM CPLEX [41] on a computer laptop equipped with 2.80 GHz Intel CPU and 8GB of RAM. To express the all parameter of the system in...
per-unit, the power base of the test system is set at 10MVA. The voltage base of the system is set at 12.66kV at utility side. The other details of MGs can also be found in [40] including output capacity, price information, and load capacity. The following cases are used for experiments:

Case 0: Grid-connected MGs in a deterministic case (46)
Case 1: Grid-connected MGs in worst case scenarios (52)
Case 2: Islanded mode of MGs operation

Fig.2. Modified IEEE 33-bus distribution system

Case 0 and Case 1: The goal is to find the optimal bidding/offering strategy within a 24-hour time horizon. Fig.3 shows DLMP trend over time for both cases, in which the same DLMP is applied to all nodes in the network at a specific time. We find very little variation in DLMPs between nodes, reflecting a lack of binding line constraints on this small network. The trend shows different DLMPs between the deterministic model and the robust model during 1:00am-13:00pm and 20:00pm-midnight, which is referred to off-peak hours. During these specific time periods, the DLMP of the robust model is 10% higher than that of the deterministic model. There are two main reasons for this difference. First, DS-owned DGs attempt to increase offer prices to secure maximum profits because they are not sure about the real-market price. Second, some DS-owned DGs (DG1, DG2 and DG5) are not fully dispatched during off-peak hours that MGs’ bids/offers have limited influence on the market price. It also shows that DLMPs stay relatively low during the non-peak hours (less than 0.66 $/p.u.).

The DLMPs start increasing at 13:00pm until they reach the peak at 17:00pm, and gradually decline for the rest of the period. During this time period (peak-hours), the prices are considerably higher than non-peak hours. This is because the consumer requirements increase rapidly during this period, which is indicated in Table III [40]. We noticed that the DLMPs for both cases remain identical between 13:00pm and 20:00pm. After DS-owned DGs reach the maximum output capacity, the DS begins to import more power from MGs with extra generation capacity. This action helps DS to stabilize the DLMPs at the beginning and end of peak-hours. After both DGs and MGs reach the maximum capacity, the utility side is the only power supplier option that DS have, even at a relatively high price. The utility prices are the same for both cases, which provides another reason that the DLMPs are identical during peak hours.

We continue our discussions using Table I, which shows the comparison between the two cases. The results in column “Entity” are associated with DS clearing market mechanism (DS) and MG operation (MG%). The sources of DS clearing market include MGs, DGs, loads, and utility. The sources of MG operation cost consist of (1) interaction with the DS and (2) power generation. The negative values in column “Cost” indicate profits. A positive value in column “Power Injection” indicates the total power transfer from a source to an entity, while a negative value indicates the opposite direction of the power transfer. The evidence of MGs’ schedule adjustment can be found to show that MGs are helpful in dealing with DS price uncertainty during the peak hours. The power generation cost of MG1 and MG2 in the robust model ($10000 and $5300) is higher than in the deterministic model ($8900 and $4100). It is obvious that these extra powers are transferred to DS, as the difference power injection values show.

Unlike other MGs, MG3’s power generation cost ($6900 to $6400) decreases as well as power exportation (0.612p.u. to 0.514p.u.) in the robust model. There are two explanations. First, the conservative DGs’ marginal cost in the robust model results in reduction of power generated by all DS-owned DGs. To overcome the resulting power shortage, the power injection from utility increases, which can increase the total feeder loss. At the same time, the power injection of MG1 and MG2 increases to create a counter flow on the main feeder to decrease the main feeder loss. The output of the MG3 is diminished because an increasing power injection from MG3 leads to increase the feeder flow and then its ohmic losses. Most of the feeder loads can acquire power from a much closer power supply (MG1 and MG2) to reduce loss on the main feeder. This makes the MG3 not competitive. Second, as input data (Table VI. [40]) shows, MG3 has a DG (DG3) which has the least operation cost (0.03 $/p.u.) among all DGs in MGs. This DG is fully dispatched for 24 hours in both cases. As a result, the MG3 is less price sensitive than the other two MGs, which makes it less influenced by price uncertainty.

Fig. 3. DLMPs of Case 1 and Case 2

TABLE I. RESULTS COMPARISON

<table>
<thead>
<tr>
<th>Entity</th>
<th>Sources</th>
<th>Case 0:</th>
<th>Case 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost ($)</td>
<td>Power Injection (p.u.)</td>
<td>Cost ($)</td>
</tr>
<tr>
<td>PCC(MG1)</td>
<td>-106</td>
<td>-0.15</td>
<td>1216.5</td>
</tr>
<tr>
<td>PCC(MG2)</td>
<td>-3114.9</td>
<td>-0.803</td>
<td>-1641.2</td>
</tr>
<tr>
<td>PCC(MG3)</td>
<td>3818.7</td>
<td>0.621</td>
<td>3286.2</td>
</tr>
<tr>
<td>DG</td>
<td>22200</td>
<td>5.831</td>
<td>22200</td>
</tr>
<tr>
<td>Utility</td>
<td>1900</td>
<td>0.88</td>
<td>2300</td>
</tr>
<tr>
<td>Loads</td>
<td>-37500</td>
<td>-6.379</td>
<td>-32600</td>
</tr>
<tr>
<td>Total</td>
<td>-12802</td>
<td>0</td>
<td>-5239</td>
</tr>
<tr>
<td>MG1</td>
<td>PCC</td>
<td>106</td>
<td>0.15</td>
</tr>
<tr>
<td>DG</td>
<td>8900</td>
<td>2.01</td>
<td>10000</td>
</tr>
</tbody>
</table>
The total profit of DS in the robust model ($5239) is less than that in the deterministic model($12802). The decrease of profit comes from two parts. First, DS extracts more power from MGs to compensate power shortage, which is caused by price uncertainty. Second, the customers decrease their bids to acquire power from DS, which leads to profit loss from supplying load.

Table II. illustrates some examples of revised MGs bidding/offering strategy. For a specific MG $j$-th at time period $t$ : The DLMP ($\lambda_j^p$) and marginal cost ($\beta_j$) and the power exchange between MG and DS ($P_{PCC}$) are given. If we can compare the original bid/offers ($\alpha_j'$) and the adjusted bid/offer ($\alpha_j$). The MG1 at 10:00am extracts power from DS. $\beta_{1,10} > \lambda_{1,10}^p$, the bid with decreasing price is 0.44+ε. For MG2 at 21:00pm, $\beta_{2,21} < \lambda_{2,21}^p$, the fix offer is 0.5 to maintain its profit. For MG2 at 14:00pm, $\beta_{2,14} < \lambda_{2,14}^p$, the increasing offer is 0.66-ε. For MG3 at 3:00am, the $\beta_{3,3} > \lambda_{3,3}^p$, the fixed offer price is set at 0.5.

Table III. is the results comparison between the original and the modified bidding/offering strategies for DS and MGs operation cost. The modified example assumes that the price interactions remain the same when the trading prices are modified. The MGs benefit from the policies to maintain the profitability. In the meantime, the obtained market clearing profit of DS decreases correspondingly. It can be seen that there is a $470.08 total cost saving for MGs and a $297.69 profit loss for DS. Therefore, we can expect that the proposed policy is practical and incentive, especially in the infancy of MG industry deployment.

### Table II. Example of Modified MG Bidding/Offering Strategy

<table>
<thead>
<tr>
<th>$j$</th>
<th>$t$</th>
<th>$\lambda_j^p$ (S/ p.u.)</th>
<th>$P_{PCC}$ (S/ p.u.)</th>
<th>$\alpha_j'$ (S/ p.u.)</th>
<th>$\beta_j$ (S/ p.u.)</th>
<th>$\alpha_j$ (S/ p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.44</td>
<td>-0.024</td>
<td>0.44</td>
<td>0.5</td>
<td>0.44+ε</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>0.91</td>
<td>-0.047</td>
<td>0.91</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>0.66</td>
<td>0.026</td>
<td>0.66</td>
<td>0.5</td>
<td>0.66-ε</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.44</td>
<td>0.022</td>
<td>0.44</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Table III. Comparisons of MG Bidding/Offering Strategy

<table>
<thead>
<tr>
<th>Entity</th>
<th>Original</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>MG1</td>
<td>8783.5</td>
<td>8765.80</td>
</tr>
<tr>
<td>MG2</td>
<td>6941.20</td>
<td>6796.45</td>
</tr>
<tr>
<td>MG3</td>
<td>3286.20</td>
<td>2978.57</td>
</tr>
<tr>
<td>Total MGs</td>
<td>19010.90</td>
<td>18540.82</td>
</tr>
<tr>
<td>DS</td>
<td>-5238.50</td>
<td>-4940.81</td>
</tr>
</tbody>
</table>

**Case 2:** This case studies the special occasion that MGs switch working mode from grid-connected to islanding in case of contingencies. We use $T - \tau$ islanding rules [9] to test the system. By introducing binary variable $\rho_j$ with constraints (55), (56) in upper level programming model, the $\rho_j$ can control the MG working modes switch. Then we add one more constraint (57) to control the total number of MG islanding hours.

$$\rho_j P_{PCC}^{\min} \leq P_j \leq \rho_j P_{PCC}^{\max} \forall t, \forall i \in J$$ (55)

$$\rho_j Q_{QCC}^{\min} \leq Q_i \leq \rho_j Q_{QCC}^{\max} \forall t, \forall i \in J$$ (56)

$$\sum_j \rho_j = T - \tau \forall i \in J$$ (57)

The total number of islanding-hours ($\tau$) from zero up to eight hours is tested based on the total operation time (24-hour ($T$) in the deterministic model). Fig. 4 and Fig. 5 show that the operation cost for MGs and clearing market profit for DS remain relatively stable as islanding hours increase. The larger number of islanding-hours results in decreasing interactions between MGs and DS. For MG with enough reserve, the more power generated through its own DGs to compensate the power lost during the islanding-hour. If not, the load shedding process is needed, which is likely to increase operation cost for MG. The DS, on the other side of islanding event, reacts to islanding events correspondingly. DS-owned DGs react to MGs’ islanding action with an increasing or decreasing power output schedule. The solution results show that each MG has enough operating reserve to supply its local load without load shedding. Therefore, Fig. 4 and Fig. 5 illustrate that the MG’s operation cost and DS’s clearing market profit depend on their DGs’ marginal cost in different number of hours islanding mode.

**Fig. 4.** Operation cost of MGs with increasing islanding time

**Fig. 5.** DS clearing market benefits

In summary, the coordinated pool strategy provides an efficient way for MGs to participate into DEM with lower cost. A bidding/offering strategy enables MGs to successfully help DS to handle price uncertainty and islanding.
V. CONCLUSION
A coordinated pool strategy for microgrid as price maker to participate in market-based distribution electricity market was proposed and formulated. We presented a reformulation of the original bi-level model as a linear mixed-integer programming model, which is easier to solve. Three sets of experiments (models, strategies, and configurations) were performed to compare (1) deterministic model vs. robust optimization model, (2) original strategy vs. revised strategy, and (3) islanding mode vs. non-islanding mode. It was shown that having MGs in DS can help stabilize the DLMPs during peak hours, and mitigate impact when an MG runs in an islanding mode. It is also shown that the proposed coordinated pool strategy performed well in dealing with the interactions between the DS and MGs. Furthermore, the market-based DEM created a fair and competitive environment for all market participants. Utilizing MGs as price makers with associated market risk enabled MGs to become competitive through a bi-directional power flow. One can extend our model to include ancillary service market.

VI. APPENDIX
Derivation of Robust Objective Function (50)
\[
\max_{\delta_{it}^l, \delta_{it}^U, \delta_{it}^L} \left( \sum_{i} \delta_{it}^l + \delta_{it}^U \varepsilon \right) M_{it}^p - \sum_{i} \left( \delta_{it}^l + \delta_{it}^U \varepsilon \right) \theta_{i} \theta_{it}^p \right)
\]
\[
= \max_{\delta_{it}^l, \delta_{it}^U, \delta_{it}^L} \left( \sum_{i} \left( \delta_{it}^l + \delta_{it}^U \varepsilon \right) M_{it}^p \right) - \min_{\delta_{it}^l, \delta_{it}^U, \delta_{it}^L} \left( \sum_{i} \left( \delta_{it}^l + \delta_{it}^U \varepsilon \right) \theta_{i} \theta_{it}^p \right)
\]
\[
= \sum_{i} \left( \delta_{it}^l + \delta_{it}^U \varepsilon \right) M_{it}^p - \sum_{i} \left( \delta_{it}^l - \delta_{it}^U \varepsilon \right) \theta_{i} \theta_{it}^p
\]

REFERENCES


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