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A Multi-objective MPEC Model for Disaster Management of Power System Restoration

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Abstract

Restoration of a power system network following a disaster or an extreme event is an urgent action. This process can be carried out through sectionalization of the power grid. The system sectionalization consists of determining the proper disjoint points to divide the entire blackout area into several sections. Then, in each section the electrical loads could be supplied by emergency power resources called “black-start” generation units for disaster management. In this study, to find the optimal sectionalization set, three critical objectives are minimized: load shedding, restoration time, and the cost of power generation. The proposed model is composed of two levels: an upper level and a lower level. The upper level model is “network sectionalization” which includes a set of innovative mixed integer linear constraints while the lower level model is “electrical loads energizing”; in which the restoration of all sections is conducted at the same time. A novel mathematical programming with equilibrium constraints (MPEC) solution methodology and pre-emptive programming (PEP) are both presented to solve the proposed multi-objective MPEC model. The efficiency of model is examined by two case studies 6- and 118-bus IEEE test systems. Promising numerical results are reported.

Keywords
Disaster management, mathematical programming with equilibrium constraints (MPEC), multi-objective model parallel restoration, power network, sectionalization.

Nomenclature

Sets			
g	Index for generators/sections, g={1, ..., NG}	ς_i / ξ_d	Cost rate of restoration of bus i/demand d
i, j	Index for buses, i ={1, ..., NB}.	c_g	Cost of power generation at unit g
d	Index for demand loads, d={1, ..., ND}	**KG**	Incident matrix of generation units
t	Index for time, t={1, ..., NT}	**KL**	Incident matrix of transmission lines
l	Index for transmission lines, l={1, ..., NL}	**KD**	Incident matrix of demands
Parameters		**Variables**	
$P_{g,\min}$ / $P_{g,\max}$	Maximal and minimal generating capacity of generation unit g.	P_{gt}	Generated power of unit g at time t
RU_g / DU_g	Ramp up/down rate of generation unit g	LS_{gt} / $\widetilde{LS}_{dt}$	Load shedding of section g/demand d at time t
D_{it} / $\widetilde{D}_{dt}$	Load demand on bus i/demand d at time t	s_{ig}	State of bus i at section g
$PL_{l,\max}$	Power line capacity of line l	PL_{lt}	Power flow on line l at time t
x_l	Reactance of line l	θ_{it}	Phase angle of bus i at time t
a_{ij}	State of connection between bus i and j	T_{ig}	total restoration time of bus i in section g
U_l	State of line l	T_d^{Load}	Load pick up time of demand d
$VOLL$	Value of lost load equal to 1000 \$/MWh	ψ_t	Auxiliary current time equal to t at time t.

1. Introduction
The severity of disaster damage varies from year to year, but disasters often cause power outages that have considerable influences on both quality of life and social economic. Long outages have been reported recently which

are caused mostly by extreme weather such as storms and hurricanes. Hurricane Ike was one the worst disasters that occurred in 2008 which was the third-costliest among Atlantic hurricanes with $25 billion damages. Electricity failure was a terrible consequence of hurricane Ike, and this costly failure took weeks to be restored [1]. Substantial human and financial assets are always spent to prepare for aggressive disasters, and recover from them. Therefore, industrial engineers work together with power system engineers to make the crucial decisions corresponding to how resources are allocated for preparation and recovery of a power system. Unfortunately, due to the complex nature of electrical grid, these provision and recovery procedures are imperfect by the expertise and intuition of power engineers. Repair or replacement of failed facilities is usually a long procedure, therefore, it is required to run an emergency restoration before. In a disaster management plan, the system will be enabled to provide the critical loads by available components and emergency power generation units. These generation units are called black-start (BS) units. Restoration through BS units is performed with a sectionalization approach to build the network within the separated sections and be prepared to reconfigure the network when the damaged components are back [2].

Sectionalization is a build-up approach that could restore a large area in a minimum time period if the appropriate sectionalization plan is chosen. In a deterministic sectionalization, the post disturbance status of a power system is assumed to be available. Also, the target area includes the critical loads to be determined [2]. Afterwards, a plan should be provided to rebuild the transmission network. The main objective of a sectionalization plan for power system restoration is minimization of unserved load [2, 3]. The restoration time is also minimized to restore the system as quickly as possible [4]. Since the amount of unserved load and the restoration time are critical in a restoration plan, this study focuses on both of them to be minimized. In this order, heuristic algorithms and mathematical models have been developed [5, 6]. Due to the nature of this problem, respected mathematical models are mixed integer programming (MIP) with binary sectionalization sets and binary components' state variables. Hence, without any modifications, solving the model for large scale networks is pretty complicated. Model decomposition is a common approach to reduce the complexity of model's solution. For instance, a bi-level programming (BLP) approach has been taken to model sectionalization and restoration within two levels [4]. The BLP approach can be solved through iterative solution methodology or directly by mathematical program with equilibrium constraints (MPEC) [7].

MPEC solution methodology is a common approach in strategic electricity market problems in power system area [8, 9] in which the upper level model is the decision making model and the lower level is the market clearing model. The other application of MPEC in this area is market-based power system maintenance scheduling that can be modeled as a BLP with revenue optimization in the upper level model and market clearing in lower level model [10]. A power system restoration model can be formulated with BLP [4] and since the lower model is a linear programming (LP), here, the MPEC approach is applied to recast the MIP model as explained in Section 2. MPEC approach finds the solution by introducing a new model based on strong duality theory and integration of the two models. The new integrated model is the upper level model with the Karush–Kuhn–Tucker (KKT) conditions of the lower level model. The equivalent MPEC of a model is a non-linear programming (NLP) because of the complementary slackness conditions. Hence, this study solves the restoration model through an equivalent linearized MPEC model. Additionally, since the proposed MPEC model is a multi-objective mathematical model, it is solved through a goal programming approach, called pre-emptive programming (PEP) [11].

The main contributions and novelty of this study are: formulating an MPEC to derive the optimum sectionalization for power system restoration and solving a multi-objective model in this order. This paper is organized as follows: the model description is presented at Section 2. Section 3 describes the solution methodology. The numerical results on 6- and 118 bus IEEE test systems are shown in Section 4, and the paper is concluded in Section 5.

2. Model description

Power system restoration minimizes the load shedding and restoration time subjected to three sets of constraints: the physical constraints, the sectionalization constraints, and the line state constraints. The physical constraints includes power generation limitation (1), ramp-up (2), ramp-down (3), and sections' load balance constraints (4):

$$P_{g,\min} \le P_{gt} \le P_{g,\max}, \forall g, \forall t \tag{1}$$

$$P_{gt} - P_{g(t-1)} \le RU_g, \forall g, \forall t \tag{2}$$

$$P_{g(t-1)} - P_{gt} \le RD_g, \forall g, \forall t \tag{3}$$

$$P_{gt} + LS_{gt} = \sum_{i=1}^{NB} s_{ig} D_{it} \ , \forall g, \forall t \tag{4}$$

The sectionalization constraints vary based on consideration of the power system network. Excluding the network in the sectionalization process might lead to many disconnections inside sections that make it impossible to fully restore those sections. On the other hand, the grid connectivity imposes complexity in assigning buses to sections and requires more constraints, e.g., line power flow constraints. To reduce the amount of load loss, a network-based sectionalization is brought into this study, and line power flow constraints are added to the physical constraints as follows:

$$\left| PL_{lt} \right| \le PL_{l,\max} \ , \forall l, \forall t \tag{5}$$

$$PL_{lt} = (\theta_{it} - \theta_{jt}) / x_l \ , \forall l \sim (i,j), \forall t \tag{6}$$

$$\theta_{ref} = 0 \tag{7}$$

Sectionalization is the assignment of grid's buses to sections. The number of sections assumed to be given is equal to the number of BS generation units [4]. Here, the assignment is performed according to the restoration time of a bus by a BS unit. Therefore, an initial restoration time matrix $\mathbf{T}^o = [t_{ig}^o]_{NB \times NG}$ is determined. Each element of this matrix (t_{ig}^o) includes: (i) all inevitable delays between bus *i* and BS unit *g* that is considered to be given [4] and (ii) load pick up times of BS unit *g*. By this definition, the main sectionalization constraints is the assignment constraint (8) [4]. The following two constraints guarantee the assignment of each bus to exactly one section (9) and prevent any empty section (10).

$$T_{ig} - t_{ig}^o \cdot s_{ig} - M(1 - s_{ig}) \le 0 \ , \forall i, \forall g \tag{8}$$

$$\sum_{g=1}^{NG} s_{ig} = 1 \ , \forall i \tag{9}$$

$$\sum_{i=1}^{NB} s_{ig} \ge 1 \ , \forall g \tag{10}$$

To push the model to form interconnected sections, another constraint is required which implies assignment of a bus to a section only when it has at least one connection within that section (11) [12].

$$s_{ig} \le \sum_{j=1}^{NB} s_{ig} \cdot a_{ij} \ , \forall i, \forall g \tag{11}$$

The line state constraint set (12) is determining the dis-joint transmission line which are found upon the selected sectionalization pattern.

$$U_l = \sum_{g=1}^{NG} s_{ig} \cdot s_{jg} \ , \forall l \sim (i,j) \tag{12}$$

This constraint set is nonlinear which makes the model complicated. In order to solve the model at lower complexity, a decomposition can be conducted to first find the sectionalization solution in a model without network consideration. Afterwards, the optimal power flow could be found in a second model. Therefore, the line state constraints would be extracted from the model to be calculated offline before solving the second model.

As a result, the first level model minimizes the total load shedding at each section and the restoration time (13). Constraints (1)-(3), (8)-(11) restrict the solution area at this level. The first model is upper level or "network sectionalization" model.

$$\min_{LS,T,P,s} \sum_{t=1}^{NT} \sum_{g=1}^{NG} VOLL \cdot LS_{gt} + \sum_{i=1}^{NB} \sum_{g=1}^{NG} \varsigma_i T_{ig} \tag{13}$$

$$\min_{LS,T,P,PL} \sum_{t=1}^{NT} \sum_{d=1}^{ND} VOLL \cdot \widetilde{LS}_{dt} + \sum_{d=1}^{ND} \xi_d T_d^{Load} \tag{14}$$

The second level is the lower level or "electrical loads energizing" that minimizes the load shedding of each load bus as well as restoration time (14) and is subjected to constraints (1)-(3) and (5)-(7). In order to find the load pick up time and also reflect it on restoration time of each load bus, the following constraint set is added:

$$T_{dt}^{Load} \ge \psi_t + M \cdot \widetilde{LS}_{dt} \ , \forall d, \forall t \tag{15}$$

The load balance constraint in the second level model is shown by equation (16) that provides the load balance on each bus while the load balance constraint of the first level (4) was on each section.

$$\sum_{g=1}^{NG} KG_{ig} P_{gt} + \sum_{l=1}^{NL} KL_{il} PL_{lt} + \sum_{d=1}^{ND} KD_{id} L\tilde{S}_{dt} = \sum_{d=1}^{ND} KD_{id} \tilde{D}_{dt} \quad ,\forall i, \forall t \tag{16}$$

Another objective of the proposed model in the both upper and lower level models is cost of generation (17). Minimization of power generation cost besides load shedding, motivates the model to find a solution at lower cost.

$$\sum_{t=1}^{NT} \sum_{g=1}^{NG} c_g P_{gt} \tag{17}$$

In general, there are some different approaches to deal with a bi-level model such as iterative optimization algorithm (IOA) and MPEC. However, according to the literature [12] MPEC can find better solution for power grid restoration model than IOA if its complexity get reduced to be solved faster. In order to present the equivalent MPEC of the restoration model, the second level model can be rewritten to minimize $\mathbf{Cx}$ while $\mathbf{x}$ is a matrix of all variables. All the inequality constraint sets of this model, (1)-(3), (5), and (15) can be rewritten in the form of $\mathbf{Ax} \le \mathbf{b}$ and the equalities, (6), (7), and (16) can be drafted as $\mathbf{E}x = \mathbf{f}$. As a result, the KKT condition of this model would be as presented in (18), where $\boldsymbol{\mu}$ is the dual variable regarding the inequality constraint and $\boldsymbol{\upsilon}$ is dual variable of equality constraint.

$$\mathbf{C} - \boldsymbol{\mu}^T \mathbf{A} - \boldsymbol{\upsilon}^T \mathbf{E} = \mathbf{0} \tag{18a}$$

$$\boldsymbol{\mu}^T \cdot (\mathbf{b} - \mathbf{Ax}) = \mathbf{0} \tag{18b}$$

$$\mathbf{Ex} = \mathbf{f} \tag{18c}$$

$$\boldsymbol{\mu} \le \mathbf{0} \tag{18d}$$

The MPEC model objective is the same as the upper level objective (13) subjected to the upper level constraints and the lower level KKT conditions (18). Hence, the MPEC solution ensures the optimality of the lower level model as well as the upper level model. To reduce the complexity of MPEC model, the non-linear complementary slackness condition (18b) is replaced by the following linear constraints, where $\mathbf{z}$ is a binary vector, and $\dot{\mathbf{M}}$ and $\ddot{\mathbf{M}}$ are two parameters with a large enough value to relax respected constraints according to the value of auxiliary variable $\mathbf{z}$ [13].

$$\mathbf{b} - \mathbf{Ax} \le -\mathbf{z}^T \cdot \dot{\mathbf{M}} \tag{19}$$

$$\boldsymbol{\mu} \ge (1 - \mathbf{z}) \cdot \ddot{\mathbf{M}} \tag{20}$$

3. Solution Methodology

The objectives of the proposed model include load shedding, restoration time, and cost of generation. There may be conflicts between these two terms such as decreasing load shedding which can enhance cost of generation and vice versa. Furthermore, these three terms can have different scales. In this order, pre-emptive goal programming (PEP) could find an optimal point which guarantees the optimality of all terms at the same time.

To solve the proposed MPEC model with PEP, the model is solved to optimize the first priority term in the feasible area of the model. For the next term, a new variable (ε_1) is added to the second priority term to keep the optimality of the first term. A new constraint (21) is also introduced to support the first term optimality. The same steps is required to consider the term with the lowest priority [11]. In this study, due to the terms' criticality, one can decide load shedding as the first, then restoration time, and finally cost of generation.

$$\sum_{t=1}^{NT} \sum_{g=1}^{NG} VOLL \cdot LS_{gt} \le \sum_{t=1}^{NT} \sum_{g=1}^{NG} VOLL \cdot LS_{gt}^{*,1} + \varepsilon_1 \tag{21}$$

4. Numerical Examples

This section examines the performance of the proposed multi-objective MPEC model by two case studies: 6-bus IEEE test system as a small scale case and 118-bus IEEE test system which is a large scale case [14]. Both of these experiments are implemented using CPLEX 12.3.0.0 under GAMS 24.4.5 on a PC with Intel Xeon 2.53GHz, 12-core, and 128GB of RAM. It is assumed that the post-disturbance states of cases are given based on a predicted extreme weather and the given fragility of transmission lines which gives a set of probably failed lines. Also, the model is supposed to find the best switching pattern in order to sectionalize these networks and perform an emergency restoration for each of them. The 6-bus test system is given with two BS units on bus 1 and 6. The BS units' properties are presented at Table 1. The transmission line data and the load demands are as defined in IEEE standard test systems.

The loads on bus 3 and 4 are assumed to be critical loads. Noted that in a real case the criticality of loads are given by the power grid owner or the decision maker.

Table 1: 6-bus IEEE test system BS units

Unit #	Bus #	Min (MW)	Max (MW)	Ramp up/down (MW/h)
1	1	100	220	55
2	6	10	100	50

The model solution gives the 6-bus sectionalized grid as presented in Fig. 1. In the optimal sectionalization, BS unit 1 provides the demand on bus 4 and BS unit 2 provides the demands on buses 3 and 5. BS unit 1 and line 1-4 both have nearly enough capacity to satisfy bus 4's demand, except at pick load hours which caused 0.03% load shedding in this section. BS unit 2's maximum power generation limits the demand's satisfaction on section 2 to 100 MW, therefore, 62% load is lost while the critical load on bus 3 is 100% provided.

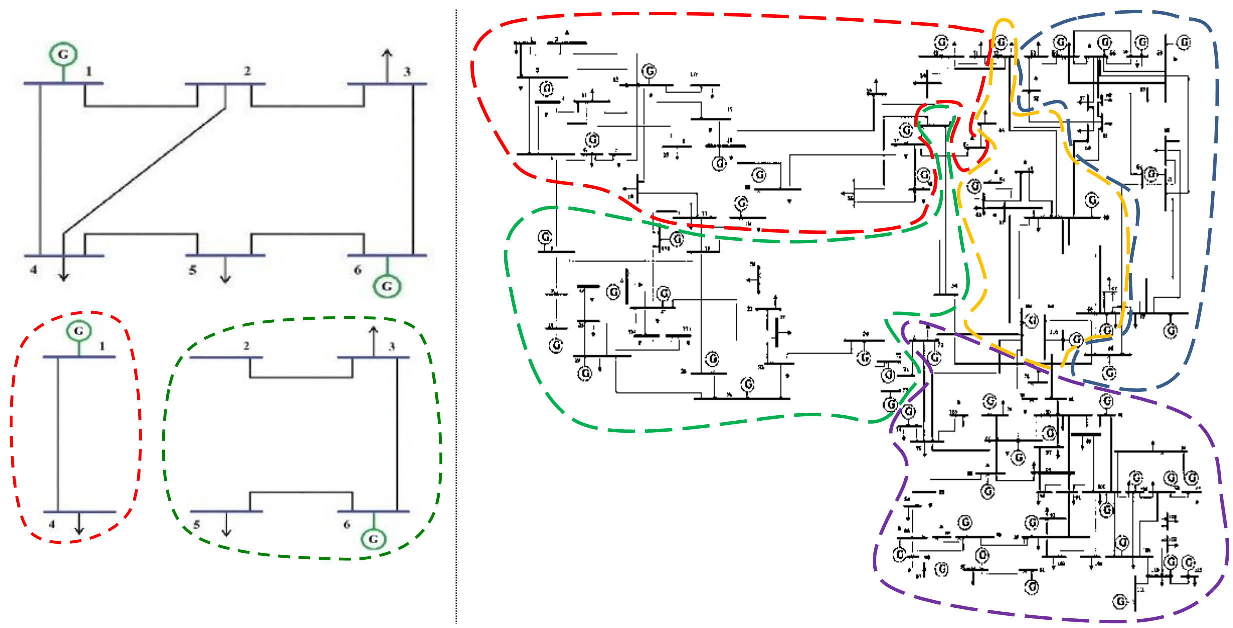

Figure 1: 6-bus IEEE test system before/ after sectionalization and 118-bus IEEE test system after sectionalization

Figure 2 illustrates the hourly critical demands as well as the total demands versus the served load in the optimal 6-bus sectionalized grid. Although there is some distance between the total demands and the served load curves, all critical loads in this case has been provided by the proposed emergency restoration as fast as 7.55 hrs. Line availability also represents the robustness of the sectionalized grid that is equal to 57% in small scale case [15].

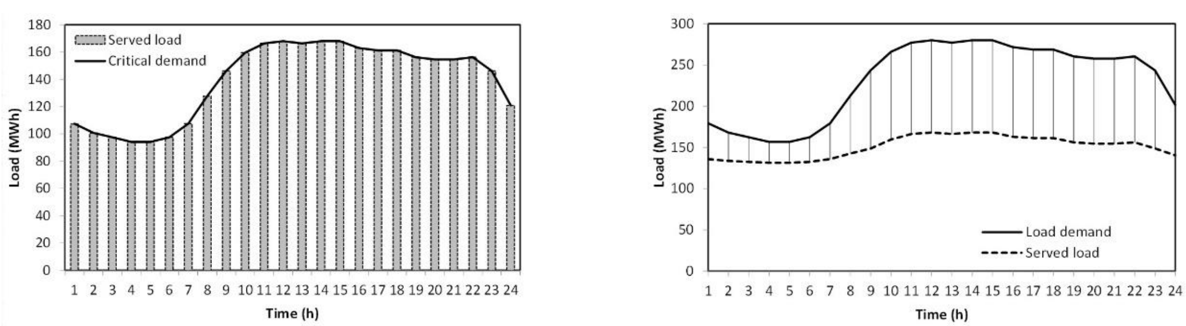

Figure 2: 6-bus IEEE test system's load satisfaction

The 118-bus case study is assumed to have five fast-response BS units on buses 12, 25, 49, 66, and 100 with same power generation capacity of 2000 MW. The results of both cases have been summarized in Table 2 which shows fully restoration of critical loads within 9.16 hours in 118-bus case study while there is 26% load shedding in total loads. The resulted grid depicted by Fig. 1 is a robust and well-connected network in the view of fact that 75% of transmission lines are available after sectionalization [15]. The CPU time of both cases, 1.25 and 11.232 seconds, shows the efficiency of the model.

Table 2: Post-restoration results of small and large scale case studies

	Load shedding %		Critical load restoration time (h)	Line availability %	CPU time
	Critical load	Total load			
Small scale case	0.02%	37%	7.55	57%	0:00:01.250
Large scale case	0.00%	26%	9.16	75%	0:00:11.232

5. Conclusion

A linear MPEC model is proposed for a power system restoration. The restoration is performed by a network-based sectionalization. The proposed model is a multi-objective model to minimize the load shedding as well as the restoration time. The simulation results confirm the efficiency of the model for both small and large scale case studies. Rather than the computation time, the sectionalization results in 100% restoration of critical loads in both cases. Both small and large scale case studies are restored in 7.55 and 9.16 hours, respectively, which are low enough following an extreme blackout. The model is open to consider the uncertainty of a system's state following a disruption as a future work.

References

1. Berg, R., 2009. Tropical cyclone report: Hurricane Ike (al092008), 1-14 September 2008. National Hurricane, Center.
2. Fink, L.H., Liou, K.L. and Liu, C.C., 1995. From generic restoration actions to specific restoration strategies. IEEE Transactions on power systems, 10(2), pp.745-752.
3. Ancona, J.J., 1995. A framework for power system restoration following a major power failure. IEEE Transactions on power systems, 10(3), pp.1480-1485.
4. Liu, W., Lin, Z., Wen, F., Chung, C.Y., Xue, Y. and Ledwich, G., 2015. Sectionalizing strategies for minimizing outage durations of critical loads in parallel power system restoration with bi-level programming. International Journal of Electrical Power & Energy Systems, 71, pp.327-334.
5. Hsiao, Y.T. and Chien, C.Y., 2000. Enhancement of restoration service in distribution systems using a combination fuzzy-GA method. IEEE Transactions on Power Systems, 15(4), pp.1394-1400.
6. Nagata, T., Sasaki, H. and Yokoyama, R., 1995. Power system restoration by joint usage of expert system and mathematical programming approach. IEEE Transactions on Power Systems, 10(3), pp.1473-1479.
7. Luo, Z.Q., Pang, J.S. and Ralph, D., 1996. Mathematical programs with equilibrium constraints. Cambridge University Press.
8. Kazempour, S.J., Conejo, A.J. and Ruiz, C., 2011. Strategic generation investment using a complementarity approach. IEEE Transactions on Power Systems, 26(2), pp.940-948.
9. Baringo, L. and Conejo, A.J., 2012. Transmission and wind power investment. IEEE transactions on power systems, 27(2), pp.885-893.
10. Pandzic, H., Conejo, A.J. and Kuzle, I., 2013. An EPEC approach to the yearly maintenance scheduling of generating units. IEEE Transactions on Power Systems, 28(2), pp.922-930.
11. Ignizio, J.P., 1982. Linear programming in single-& multiple-objective systems. Prentice Hall.
12. Abbasi, S., Barati, M., and Lim, G., 2017, A Parallel Sectionalized Restoration Scheme for Resilient Smart Grid Systems, IEEE Transactions on Smart Grid (submitted).
13. Karoonsoontawong, A. and Waller, S., 2007. Robust dynamic continuous network design problem. Transportation Research Record: Journal of the Transportation Research Board, (2029), pp.58-71.
14. "Power Systems Test Case Archive" [Online], Available: http://www2.ee.washington.edu/research/pstca/
15. Beygelzimer, A., Grinstein, G., Linsker, R. and Rish, I., 2005. Improving network robustness by edge modification. Physica A: Statistical Mechanics and its Applications, 357(3), pp.593-612.