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A Decomposition Algorithm for the Two-Stage Chance-Constrained Operating Room Scheduling Problem

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ABSTRACT The required time for surgical interventions in operating rooms (OR) may vary significantly from the predicted values depending on the type of operations being performed, the surgical team, and the patient. These deviations diminish the efficient utilization of OR resources and result in the disruption of projected surgery start times. This paper proposes a two-stage chance-constrained model to solve the OR scheduling problem under uncertainty. The goal is to minimize the costs associated with OR opening and overtime as well as reduce patient waiting times. The risk of OR overtime is controlled using chance constraints. Numerical experiments show that the proposed model provides a better trade-off between minimizing costs and reducing solution variability when compared to two existing models in the literature. It is also shown that the three models converge as the overtime probability threshold approaches one. Moreover, it is observed that the individual chance constraints result in the opening of fewer rooms, lower waiting times, and shorter solution times when compared to that of joint chance constraints. A decomposition algorithm is applied that solves large test instances of the OR scheduling problem, that of which is known to be NP-hard. Strong valid inequalities are derived in order to accelerate the convergence speed. The proposed approach outperformed both a commercial solver and a basic decomposition algorithm after solving all test instances with up to 89 surgeries and 20 ORs in less than 48 minutes.

INDEX TERMS Chance Constraints, Mixed-Integer Programming, Operating Room Scheduling, Two-Stage Stochastic Programming, Uncertainty

I. INTRODUCTION

HEALTH care expenditures are expected to constitute 25% of the US gross domestic product (GDP) in 2025, an increase from 15.9% in 2005 [21]. Surgical expenses contribute to 30% of health care expenditures and are expected to grow from \$572 billion in 2005 to \$912 billion (2005-valuated dollars) in the year 2025. Surgical procedures are complex tasks requiring a variety of specialized and expensive resources. In 2011, hospitalizations involving surgical procedures constituted 29% of total hospital stays while contributing to 48% of total hospital costs in the US [33]. In light of these reports, surgeries are recognized as the most crucial activities performed in hospitals from a social, medical and economic point of view.

Several survey articles have recognized the surgery duration uncertainty as a major obstacle to developing practical and cost-effective OR schedules [18]. This paper proposes a chance-constrained programming model that: 1) provides cost-effective OR schedules by considering both deterministic and stochastic costs, 2) maintains a low OR overtime probability and compares individual and joint chance constraints, 3) results in a better cost-variability trade-off compared to two existing models in the literature and 4) solves such problems at a faster rate than the two aforementioned existing models before applying any solution algorithms. Moreover, a computationally efficient solution method and strong valid inequalities are provided to facilitate timely decision-making in the case of disruptions in the schedule.

The OR scheduling literature has been reviewed in several survey articles [2], [18], [28]. The published literature has been classified using several categories, including *uncertainty*. Variable surgery duration is one of the most commonly studied sources of uncertainty by the Operations Research community. It is shown that mitigating the impact of disruptions in the schedule due to uncertainty can lead to higher capacity utilization and lower costs [19], [20].

TABLE 1: Research Gaps in the Chance-Constrained OR Scheduling Literature

Authors (Year)	**Decisions**			**Objective**	
	OR	**Resource Allocation**	**Sequencing & Scheduling**	**Deterministic Costs**	**Stochastic Costs**
Shylo et al. (2012)		✓			OR Idle Time
Zhang et al. (2015)		✓			OR Overtime
Deng et al. (2019)	✓	✓	✓	OR Opening	
Noorizadegan & Seifi (2018)	✓	✓	✓	OR Opening & Turn-Over	
Wang et al. (2017)	✓	✓		Operational	
This Paper	✓	✓	✓	OR Opening	OR Overtime & Patient Waiting Time

Hence, it is crucial to ensure that the provided schedule works reliably in the presence of large variability in surgery durations.

Numerous works have used stochastic programming to model the uncertain surgery durations in the OR scheduling problems [1], [7], [25], [32]. The majority of these models consider optimizing the expectation of costs/revenues during the planning horizon [10]. For problems with moderate variability, using the expected value (EV) can result in desirable outputs. However, the obtained solutions may show poor performance for problems displaying frequent changes in a less predictable manner [24]. A number of articles considered using the Conditional Value-at-Risk (CVaR) [27] to account for undesirable realizations of the uncertain parameters [15], [22], [29]. The CVaR function minimizes the expected tail of costs.

Another array of articles have used the chance-constrained programming (CCP) models [3] to address the uncertainty. This approach mitigates the risk of disadvantageous events (e.g., OR overtime, patient waiting time) exceeding the specified thresholds, rather than merely minimizing their expected value [4], [13], [35]. Shylo et al. [30] applied chance-constraints to control the OR block overtime in the OR surgery planning problem. Zhang et al. [36] studied a chance-constrained OR surgery allocation problem. Deng and Shen [4] developed a two-stage stochastic model for the multi-server appointment scheduling problem with a joint chance constraint on server overtime. They applied the proposed model and solution approach to solve OR scheduling problem test instances. Jebali and Diabat [11] studied the surgery planning problem under uncertain surgery duration, length of stay in the intensive care unit (ICU), and emergency patient arrival. They employed chance constraints to control the violation of ICU capacity. Wang et al. [31] proposed a distributionally robust chance-constrained model for the surgery planning problem with stochastic surgery durations. Noorizadegan and Seifi [23] proposed a CCP model for the surgery planning problem with uncertain surgery durations. Kamran et al. (2018) [12] proposed a two-stage stochastic model with chance constraints on OR overtime for the advance scheduling problem. Deng et al. [5] developed a distributionally robust chance-constrained model for the OR scheduling problem. They control the risk of OR overtime and surgery waiting using joint chance constraints.

Table 1 summarizes the selected published literature and identifies the research gaps that are addressed in this paper. First, it can be observed that very few articles proposed a CCP model for the OR scheduling problem under uncertainty [4], [5]. Surgery scheduling problems often have a more complex structure resulting from a variety of decisions, such as OR opening, patient-to-OR assignment, surgery sequencing, and projected and actual start times before and after the realization of random surgery durations, respectively. CCP-based models have the potential to effectively handle such large variabilities in daily surgery scheduling problems [18].

Second, a majority of the models have neglected the importance of minimizing the stochastic second-stage costs. Their primary focus has been on providing schedules within the specified risk tolerances while also aiming to minimize deterministic performance measures, such as fixed OR opening costs [5], [23]. Unlike existing approaches, this paper proposes a chance-constrained model that aims to minimize both deterministic and stochastic costs for the OR scheduling problem. The significance of considering both classes of costs is highlighted using numerical experiments.

Third, we provide insightful observations about the performance of three different models (CCP, CVaR and EV) in solving the stochastic OR scheduling problem under various risk thresholds. The proposed model is compared alongside EV and CVaR models using several metrics such as total costs, OR utilization and solution time. Moreover, the performances of both individual and joint chance constraints are compared in terms of OR opening decisions, minimizing the second-stage costs and computational efficiency.

Finally a computationally efficient decomposition algorithm is applied to provide high-quality solutions for the large-scale test instances within reasonable time frames. We proposed an algorithm to derive feasibility cuts using the first-stage solutions that accelerate finding feasible solutions and the convergence speed.

II. MATHEMATICAL FORMULATION

A. PROBLEM DESCRIPTION

Let I be the set of elective surgeries and R be the set of operating rooms. The problem is to schedule surgeries over a daily planning horizon. We assume that operating rooms use the block booking policy [6]. Each OR is allocated to a surgical specialty according to the master surgery schedule. The incidence matrix $E = \{e_{ir}\}$ for all $i \in I, r \in R$ allows specific surgery-to-OR assignments. The surgery duration is random, denoted by a random vector $\delta = (\delta_1, ..., \delta_{|I|})^T \in \mathbb{R}_+^{|I|}$ where δ_i shows the random duration for surgery $i \in I$. We assume that the random surgery duration has finite and discrete support $\mathbb{S}$ for δ. The probability density of each scenario s is denoted by p_s, where $\sum_{s\in\mathbb{S}} p_s = 1$. Each realization of δ in scenario s is shown by $\delta^s = (\delta_1^s, ..., \delta_{|I|}^s)^T$. Every surgery on the daily booking list must be operated. No interruption is allowed once an operation has started. It is desired to decrease the probability of working OR overtime. This restriction is enforced using chance constraints. Each surgery must be assigned an OR and a projected start time. Our goal is to minimize the sum of fixed OR opening costs and expected costs corresponding to OR overtime and patient waiting times.

B. CHANCE-CONSTRAINED OR SCHEDULING PROBLEM

We propose a two-stage stochastic model for the daily OR scheduling problem with individual chance constraints enforcing overtime restrictions on every OR. Table 2 introduces the notation used in our model.

The first-stage problem involves deterministic decision making (i.e., OR opening and surgery case assignment) prior to the realization of uncertain surgery durations. After the uncertain parameters are revealed, the second-stage problem determines recourse actions (e.g., adjusting start times and adding OR overtime) that incur additional costs to provide meaningful schedules based on first-stage decisions. The goal is to minimize total costs as well as satisfy the chance constraints on OR overtime.

The first-stage problem (M_1) can be formulated as follows:

$$\min \sum_{r\in R} f_r u_r \tag{1}$$

$$x \in X \tag{2}$$

where $x = (u, y)$ is the vector of first-stage variables. Set X is the resulting set from the deterministic constraints (3)-(7) formulated as follows:

$$y_{ikr} \le u_r, \forall i \in I, k \in K, r \in R \tag{3}$$

$$\sum_{k,r} y_{ikr} = 1, \forall i \in I \tag{4}$$

$$y_{ikr} \le e_{ir}, \forall i \in I, k \in K, r \in R \tag{5}$$

$$\sum_i y_{ikr} \ge \sum_i y_{i(k+1)r}, \forall k \in K \setminus \{|K|\}, r \in R \tag{6}$$

$$u_r \in \mathbb{B}^R, y_{ikr} \in \mathbb{B}^{I\times K\times R}, \forall i \in I, k \in K, r \in R \tag{7}$$

Objective function (1) minimizes the total cost of opening operating rooms. Constraints (3) and (4) ensure that every surgery will be assigned to one and only one spot in an open OR during the day. Constraint (5) enforces eligible surgery-to-OR assignments. Constraint (6) determines the order of operating surgical cases in each OR. Constraint (7) enforces binary values for the first-stage decision variables.

The chance-constrained second-stage problem (M_2) is formulated as follows:

$$tp_{kr} \le tp_{(k+1)r}, \forall k \in K \setminus \{|K|\}, r \in R \tag{8}$$

$$tr_{krs} \ge tp_{kr}, \forall k \in K, r \in R, s \in \mathbb{S} \tag{9}$$

$$tr_{krs} \le tr_{(k+1)rs}, \forall k \in K, r \in R, s \in \mathbb{S} \tag{10}$$

$$\sum_k (tr_{krs} - tp_{kr}) \le w_{rs}, \forall r \in R, s \in \mathbb{S} \tag{11}$$

$$Pr\{tr_{krs} + \sum_i \delta_{is} y_{ikr} \le cap_r : \forall k \in K, s \in \mathbb{S}\}$$
$$\ge 1 - \alpha_r, \forall r \in R \tag{12}$$

$$tp_{kr}, tr_{krs}, w_{rs} \ge 0, \forall k \in K, r \in R, s \in \mathbb{S} \tag{13}$$

Constraint (8) determines the projected start time for each surgery according to the sequencing decisions. Constraint (9) ensures that each surgery starts after its projected start time. Constraint (10) is similar to (8) in that the actual start times must follow the sequencing decisions. Constraint (11) calculates the amount of waiting time in every OR per scenario. The chance constraints (12) state that the surgeries assigned to an OR must be finished during the regular hours (i.e., no overtime) with high probability. Constraint (13) enforces the non-negativity of the second-stage decision variables. The objective function of the second-stage problem is formulated in the remainder of this section.

The set $\mathbb{P}(s)$ of the first-stage solutions that are made to satisfy the chance-constrained second-stage problem is derived as follows:

$$\mathbb{P}(r,s) = \left\{ x \in X \mid \exists tp, tr : tr_{krs} + \sum_i \delta_{is} y_{ikr} \le cap_r \right\} \tag{14}$$

$$\mathbb{P}(s) = \bigcap_{r\in R} \mathbb{P}(r,s) \tag{15}$$

Proposition 1. *Let $\alpha_r|S|$ be an integer for every r. Then, chance constraints* (12) *are equivalent to:*

$$tr_{kr}^s + \sum_i \delta_{is} y_{ikr} \le cap_r + M z_{rs}, \forall k \in K, r \in R, s \in \mathbb{S} \tag{16}$$

$$\sum_{s\in\mathbb{S}} z_{rs} \le \alpha_r|\mathbb{S}|, \forall r \in R, s \in \mathbb{S} \tag{17}$$

where binary variable $z_{rs} = 1$ when the time capacity of room r is violated.

Proof. See Appendix A-A. □

TABLE 2: Sets, parameters and variables used in the model

	Symbol	Definition
Sets	I	elective surgery, $i \in \{1, ..., \|I\|\}$
	R	operating room, $r = \in \{1, ..., \|R\|\}$
	K	order of surgery appointment in OR, $k \in \{1, ..., \|K\|\}$
	$\mathbb{S}$	scenario, $s \in \{1, ..., \|\mathbb{S}\|\}$
Parameters	e_{ir}	=1 if surgery i can be assigned to OR r; 0 otherwise
	δ_{is}	duration of surgery i in scenario s
	f_r	fixed cost of opening OR r
	cap_r	operating time limit for OR r
	co_r	unit overtime cost of OR r
	cw_r	unit waiting cost for surgery i
	p_s	probability density of scenario s
	α_r	overtime probability threshold for OR r (confidence level), $\alpha_r \in (0, 1)$
	M	a sufficiently large number
Variables	u_r	=1 if OR r is open; 0 otherwise
	y_{ikr}	=1 if surgery i scheduled as k^{th} surgery in OR r; 0 otherwise
	tp_{kr}	projected start time for surgery i
	tr_{krs}	actual start time for surgery i in scenario s
	o_{rs}	OR r overtime in scenario s
	w_{rs}	total patient waiting times in OR r and scenario s
	z_{rs}	=1 if chance constraint on OR r is violated in scenario s; 0 otherwise

For each scenario $s \in \mathbb{S}$, an operation may be completed during regular hours ($z_{rs} = 0$) or may run into overtime ($z_{rs} = 1$). Therefore, the second-stage cost, $g(x, s)$, will be calculated differently in each case:

$$\begin{cases} g_r^1(x,s) = cw_r w_{rs} & z_{rs} = 0 \quad (18) \\ g_r^2(x,s) = cw_r w_{rs} + co_r o_{rs} & z_{rs} = 1 \quad (19) \end{cases}$$

where o_{rs} is a non-negative variable representing overtime. Therefore, the objective function of the second-stage problem can be formulated as follows:

$$\mathbb{E}_s\left[g(x,s)\right] = \mathbb{E}_s\left(\sum_r (1 - z_{rs}) g_r^1(x,s) + z_{rs} g_r^2(x,s)\right) \quad (20)$$

Objective function (20) minimizes the expected costs corresponding to OR overtime and patient waiting times. The deterministic equivalent formulation for the two-stage chance-constrained OR scheduling model (M_{DEF}) can be modeled as follows:

$$\min obj = \sum_{r \in R} f_r u_r + \frac{1}{|\mathbb{S}|} \sum_{r,s} (cw_r w_{rs} + co_r o_{rs}) \quad (21)$$

$$s.t. \quad (8) - (11), (13)$$

$$tr_{kr}^s + \sum_i \delta_{is} y_{ikr} \le cap_r$$
$$+ M z_{rs}, \forall k \in K, r \in R, s \in \mathbb{S} \quad (22)$$

$$tr_{kr}^s + \sum_i \delta_{is} y_{ikr} \le cap_r$$
$$+ M(1 - z_{rs}) + o_{rs}, \forall k \in K, r \in R, s \in \mathbb{S} \quad (23)$$

$$\sum_{s \in \mathbb{S}} z_r^s \le \alpha_r |\mathbb{S}|, \forall r \in R, s \in \mathbb{S} \quad (24)$$

$$x \in X, z_{rs} \in \mathbb{B}^{R \times \mathbb{S}}, \forall k \in K, r \in R, s \in \mathbb{S} \quad (25)$$

Using a big M value can lead to weak LP relaxations. Assigning a smaller value for M can help tighten the feasible region for the LP relaxation of M_{DEF}. Therefore, instead of setting a single large value for M, constraint-specific formulae used to calculate the big M values are developed for constraints (22) and (23) as shown below:

$$M_{rs} = \sum_i e_{ir} \delta_{is}, \forall r \in R, s \in \mathbb{S} \quad (26)$$

The values in (26) are valid because the total operation time in each OR does not exceed the duration of all surgical cases that can be allocated to the specific operating room.

III. SOLUTION APPROACH

This section describes a decomposition algorithm that solves the proposed model in Section II. The proposed algorithm can solve the model to optimality if the following assumptions are satisfied [16], [17]:

1) The random vector $\mathbb{S}$ has discrete and finite support. Specifically, $p_s = \frac{1}{|\mathbb{S}|}$ for $s \in \mathbb{S}$.
   We have stated this assumption in the problem description in Section 2.1.
2) Set X and $\mathbb{P}(s)$, $s \in \mathbb{S}$ are non-empty compact sets.
   Without loss of generality, we can assume that for every $s \in \mathbb{S}$, there exists a feasible first-stage solution that satisfies the chance constraints. Therefore, sets X and $\mathbb{P}(s)$ are finite sets of points that qualifies them as compact sets.
3) Set $conv(\mathbb{P}(s))$, $s \in \mathbb{S}$ have the same recession cone, i.e., there exists $C \subseteq \mathbb{R}^N$ such that $C = \left\{\theta \in \mathbb{R}^N | x + \lambda\theta \in \mathbb{P}(s); \forall x \in \mathbb{P}(s), \lambda \ge 0\right\}$ for all $s \in \mathbb{S}$, where $N := R + I \times K \times R$.

Proof. See Appendix A-B. □

4) There does not exist an extreme ray $\tilde{\theta}$ of conv(X) with $f^T\tilde{\theta} < 0$, i.e., the two-stage problem has a bounded optimal solution.

Proof. See Appendix A-C. □

Proposition 2. *Model M_{DEF} has an optimal solution (x^*, z^*) in which $\sum_s z^*_{rs} = \alpha_r|\mathbb{S}|$ for all $s \in \mathbb{S}$ [17].*

Proof. See Appendix A-D. □

We begin the decomposition algorithm by defining feasibility ($\mathbb{F}$) and optimality ($\mathbb{O}$) sets as follows:

$$\mathbb{F} = \Bigg\{ x \in X, z \in \mathbb{B}^{R\times\mathbb{S}} : \sum_s z_{rs} = \alpha_r|\mathbb{S}|, \\ r \in R, z_{rs} = 0 \Rightarrow x \in \mathbb{P}(r,s), s \in \mathbb{S} \Bigg\} \tag{27}$$

$$\mathbb{O} = \Bigg\{ (x,z,\rho) \in \mathbb{F} \times \mathbb{R}_+ : \\ \rho \geq \frac{1}{|\mathbb{S}|} \sum_{r,s} (1 - z_{rs}) g^1_r(x,s) + z_{rs} g^2_r(x,s) \Bigg\} \tag{28}$$

These sets will be approximated using feasibility and optimality cuts in the following master problem (**MP**):

$$\begin{aligned} &\min_{x,z,\rho} f^T u + \rho \\ &\sum_s z_{rs} = \alpha_r|\mathbb{S}|, \forall r \in R \\ &x \in X, z \in \mathbb{B}^{R\times\mathbb{S}}, \rho \geq 0 \\ &(x,z) \in \tilde{\mathbb{F}} \\ &(x,z,\rho) \in \tilde{\mathbb{O}} \end{aligned} \tag{29}$$

The sets $\tilde{\mathbb{F}}$ and $\tilde{\mathbb{O}}$ are the outer approximations of the feasibility (F) and optimality (O) sets, respectively. In the remainder of this section, we will derive strong valid inequalities to define $\tilde{\mathbb{F}}$ and $\tilde{\mathbb{O}}$.

A. FEASIBILITY CUTS

Two sets of subproblems are required to formulate the strong feasibility cuts: single-scenario optimization and single-scenario separation [16]. The optimization subproblem for the OR scheduling problem is formulated as follows:

$$h_{rs}(\gamma) = \min_x \left\{ \gamma x \mid x \in \mathbb{P}(r,s) \cap \bar{X} \right\} \tag{30}$$

Where $\gamma \in \mathbb{R}^N$ and $\bar{X} \supseteq X$, chosen such that $\mathbb{P}(r,s) \cap \bar{X} \neq \Phi$.

Proposition 3. *Problem (30) is feasible and has a finite optimal value if $\gamma \in \mathbb{R}^N$.*

Proof. See Appendix A-E. □

The separation subproblem can be formulated as follows:

$$\varrho_{rs}(\hat{x}) = \max_\pi \sum_k \pi^4_{krs} \left(\sum_i \tilde{d_{is}} \hat{y}_{ikr} - cap_r \right) \tag{31}$$

$$\left(\pi^1_{(k-1)rs} - \pi^1_{krs}\right) + \pi^5_{rs} \leq 0, \forall k \in \mathbb{K} \tag{32}$$

$$\left(\pi^2_{(k-1)rs} - \pi^2_{krs}\right) + \pi^3_{krs} - \pi^4_{krs} - \pi^5_{rs} \leq 0, \forall k \in \mathbb{K} \tag{33}$$

$$\pi^4_{krs} \leq co_r, \forall k \in \mathbb{K} \tag{34}$$

$$\pi^5_{rs} \leq cw_r \tag{35}$$

$$\sum_k \left(\pi^1_{krs} + \pi^2_{krs} + \pi^3_{krs} + \pi^4_{krs}\right) + \pi^5_{rs} = 1 \tag{36}$$

$$\pi^1_{krs}, \pi^2_{krs}, \pi^3_{krs}, \pi^4_{krs}, \pi^5_{rs} \geq 0, \forall k \in \mathbb{K} \tag{37}$$

Solving this subproblem to optimality returns a separating hyperplane of the form $\gamma x \geq \beta$ for all $x \in P(r,s)$. Let $\hat{x}$ be a solution to the master problem (MP). If $\varrho_{rs}(\hat{x}) > 0$ and $\hat{\pi}$ is the optimal solution, the separating hyperplane $-\sum_k \hat{\pi}^4_k \sum_i \tilde{d}_i y_{ikr} \geq -\sum_k \hat{\pi}^4_k cap_r$ cuts off $\hat{x}$ from $\tilde{\mathbb{F}}$. Therefore, we define the valid feasibility cuts as follows:

Theorem 1. *The following sets of inequalities are valid for $\mathbb{F}$:*

$$\gamma x + \sum_{i=1}^{l} \left(h_{g_i}(\gamma) - h_{g_{i+1}}(\gamma)\right) z_{g_i} \geq h_{g_1}(\gamma) \tag{38}$$

where $h_{\sigma_1} \geq h_{\sigma_2} \geq \ldots \geq h_{\sigma_{|\mathbb{S}|}}$, $G = \{g_1, g_2, \ldots, g_l\} \subseteq \{\sigma_1, \sigma_2, \ldots, \sigma_p\}$ and $h_{g_{l+1}} = h_{\sigma_{p+1}}$.

The remainder of this section presents another class of feasibility cuts derived from the solutions to the first-stage problem (M_1) or master problem (MP) that result in the violation of chance constraints (12).

Theorem 2. *The following set of inequalities are valid for $\mathbb{F}$:*

$$z_{rs} - \sum_{k \in K, i \in TB} y_{ikr} \geq 1 - |TB|, r \in R, s \in \mathbb{S} \tag{39}$$

where TB is the subset of surgeries that lead to the violation of chance constraints when they are assigned to the same OR.

Let $\hat{y_{ikr}}$ be the set of surgery-to-OR assignments obtained from solving (M_1) or (MP). Algorithm 1 generates inequalities of type (39).

Algorithm 1 Feasibility Cut Generation

input: Sets and parameters in Table 2, $\hat{y_{ikr}}$.
initialize: $v_{rs} \leftarrow 0, Cnt_r \leftarrow 0, TB \leftarrow \Phi$.
Calculate $v_{rs} = \sum_{k,i} \delta_{is} \hat{y_{ikr}} - cap_r, \forall r \in R, s \in \mathbb{S}$.
if $v_{rs} > 0, \forall r \in R, s \in \mathbb{S}$ **then**
    $Cnt_r \leftarrow Cnt_r + 1$.
    if $Cnt_r > \alpha_r|\mathbb{S}|, \forall r \in R$ **then**
        $TB \leftarrow \{i | \hat{y_{ikr}} = 1\}$.
        Add feasibility cut (39) to $\tilde{\mathbb{F}}$ in (MP).
    end if
end if

B. OPTIMALITY CUTS

In this section, we derive optimality cuts to add to $\tilde{\mathbb{O}}$. First, we formulate the dual problems for regular and overtime modes of the second-stage problem. For every $r \in R$ and $s \in \mathbb{S}$ where $z_{rs} = 0$, the regular mode dual problem is formulated as follows:

$$\nu^1_{rs}(\hat{x}) = \max_{\pi} \sum_k \pi^4_{krs} \left(\sum_i \tilde{d_{is}} \hat{y}_{ikr} - cap_r \right) \tag{40}$$

$$\left(\pi^1_{(k-1)rs} - \pi^1_{krs} \right) + \pi^5_{rs} \leq 0, \forall k \in \mathbb{K} \tag{41}$$

$$\left(\pi^2_{(k-1)rs} - \pi^2_{krs} \right) + \pi^3_{krs} - \pi^4_{krs} - \pi^5_{rs} \leq 0, \forall k \in \mathbb{K} \tag{42}$$

$$\pi^5_{rs} \leq cw_r \tag{43}$$

$$\sum_k \left(\pi^1_{krs} + \pi^2_{krs} + \pi^3_{krs} + \pi^4_{krs} \right) + \pi^5_{rs} = 1 \tag{44}$$

$$\pi^1_{krs}, \pi^2_{krs}, \pi^3_{krs}, \pi^4_{krs}, \pi^5_{rs} \geq 0, \forall k \in \mathbb{K} \tag{45}$$

Since the only difference between the regular and overtime mode is the introduction of overtime variables (when $z_{rs} = 1$), the overtime mode dual problem can be derived by replacing π with $\bar{\pi}$ and adding the dual constraints corresponding to overtime variables:

$$\nu^2_{rs}(\hat{x}) = \max_{\bar{\pi}} \quad \sum_k \bar{\pi}^4_{krs} \left(\sum_i \tilde{d_{is}} \hat{y}_{ikr} - cap_r \right) \tag{46}$$

$$s.t. \quad (41) - (45)$$

$$\bar{\pi}^4_{krs} \leq co_r, \forall k \in \mathbb{K} \tag{47}$$

The set of dual optimal solutions for the regular and overtime modes are shown by Π_{rs} and $\bar{\Pi}_{rs}$, respectively. Then, we formulate optimality subproblems for each mode. For a given $\tau \in \mathbb{R}^N$, $r \in R$ and $s \in \mathbb{S}$, we formulate the optimality subproblem for the regular mode as follows:

$$\psi^1_{rs}(\tau) = min \left\{ g^1_r(x, s) + \tau^T x : x \in \mathbb{P}(r, s) \right\} \tag{48}$$

Similarly, we formulate the optimality subproblem for the overtime mode as follows:

$$\psi^2_{rs}(\tau) = min \left\{ g^2_r(x, s) + \tau^T x : x \in X \right\} \tag{49}$$

Proposition 4. *Let dom* $\psi_{rs}(\tau) = \left\{ \tau \in \mathbb{R}^N : \psi_{rs}(\tau) > -\infty \right\}$. *There exists* $D \subseteq \mathbb{R}^N$ *where dom* $\psi^1_{rs}(\tau) =$ *dom* $\psi^2_{rs}(\tau) = D$.

Proof. See Appendix A-F. □

Proposition 5. *Let* $Q \subseteq \mathbb{S}$, $\pi_{rs} \in \Pi_{rs}$ *and* $\tau_{rs} = \sum_{i,k} \pi^4_{krs} \tilde{d}_{is}$ *for* $s \in Q$, *and* $\bar{\pi}_{rs} \in \bar{\Pi}_{rs}$ *and* $\tau_{rs} = \sum_{i,k} \bar{\pi}^4_{krs} \tilde{d}_{is}$ *for* $s \in \mathbb{S} \setminus Q$. *The following inequality is valid for* $\mathbb{O}$:

$$\rho + \frac{1}{|\mathbb{S}|} \sum_{r,s \in Q} \left(-\sum_k \pi^4_{krs} cap_r - \psi^2_{rs}(\tau_{rs}) \right) z_{rs}$$
$$+ \frac{1}{|\mathbb{S}|} \sum_{r,s \in \mathbb{S} \setminus Q} \left(-\sum_k \bar{\pi}^4_{krs} cap_r - \psi^1_{rs}(\tau_{rs}) \right) (1 - z_{rs})$$
$$\geq \frac{1}{|\mathbb{S}|} \sum_{r,s \in Q} \left(-\sum_k \pi^4_{krs} cap_r \right)$$
$$+ \frac{1}{|\mathbb{S}|} \sum_{r,s \in \mathbb{S} \setminus Q} \left(-\sum_k \bar{\pi}^4_{krs} cap_r \right) + \frac{1}{|\mathbb{S}|} \sum_{i,r,s} (\tau_{rs} \hat{y}_{ikr}) \tag{50}$$

C. DECOMPOSITION ALGORITHM

A decomposition algorithm is proposed to solve the two-stage chance-constrained OR scheduling problem. This algorithm has a similar structure to the Benders decomposition algorithm [26]. Rather than traditional Benders cuts, we use strong valid inequalities derived in Section III-A and Section III-B. Parameter ϵ in Algorithm 2 represents the upper bound on the relative optimality gap, calculated as $\frac{UB-LB}{UB}$.

Algorithm 2 Decomposition Algorithm

input: sets and parameters in Table 2, model M_{DEF}.
initialize: $LB := -\infty, UB := +\infty, \epsilon \in \left[10^{-3}, 10^{-6}\right]$
while $\frac{UB-LB}{LB} > \epsilon$ **do**
  Solve master problem (29).
  if (29) is infeasible **then**
    Stop. Original problem is infeasible.
  else
    Let $(\hat{x}, \hat{z}, \hat{\rho})$ be an optimal solution to (29).
    • $LB \leftarrow f^T \hat{u} + \hat{\rho}$.
    • Check feasibility of the second-stage problem by calling Algorithm 1 and evaluating the inequalities (38).
    if there exists violated inequalities **then**
      • Add feasibility cuts (38) and (39) to $\tilde{\mathbb{F}}$.
    else
      • $UB \leftarrow \sum_{r \in R} f_r \hat{u}_r + \frac{1}{|\mathbb{S}|} \sum_{r,s} \left[g^1_r(\hat{x}, s) + g^2_r(\hat{x}, s) \right]$
      • Add optimality cuts (50) to $\tilde{\mathbb{O}}$.
    end if
  end if
end while
output: optimal cost obj^* and decision variables $(x^*, z^*, tp^*, tr^*, o^*, w^*)$.

Theorem 3. *Algorithm 2 converges to an optimal solution in finite iterations.*

Proof. See Appendix A-G. □

IV. NUMERICAL EXPERIMENTS

Test problem instances are obtained from Leeftink and Hans [14]. The instances consist of different surgical specialties such as orthopedic, otorhinolaryngology, and oncology. The surgery durations follow a three-parameter lognormal distribution [9]. We used the Monte Carlo sampling method to generate a finite set of scenarios for random surgery durations. The overhead and variable costs for operating rooms are determined using the cost settings in [7]. The OR opening cost is calculated by multiplying the overhead cost by the OR available time. Each OR operates an 8-hour workday

TABLE 3: Computational efficiency of CCP, CVaR and EV

Instance	$\|I\|$	$\|R\|$	Model	Time (s)	Gap (%)
1	6	5	CCP	**1.6**	0.0
			CVaR	3.5	0.0
			EV	6.8	0.0
2	7	5	CCP	**2.2**	0.0
			CVaR	7.5	0.0
			EV	32.1	0.0
3	9	5	CCP	**11.3**	0.0
			CVaR	132	0.0
			EV	87.1	0.0
4	12	5	CCP	**140**	0.0
			CVaR	550	0.0
			EV	1258.8	0.0
5	16	5	CCP	**1169.5**	0.0
			CVaR	2623.8	0.0
			EV	3600	1.38
6	20	5	CCP	**1230**	0.0
			CVaR	3600	4.2
			EV	3600	5.02
7	23	10	CCP	**2204.3**	0.0
			CVaR	3600	11.7
			EV	3600	21.0
8	29	10	CCP	**2889.5**	0.0
			CVaR	3600	39.5
			EV	3600	41.0

and has one block that is assigned to a surgical specialty. Optimization models are implemented in Python using IBM CPLEX on a workstation with 24 cores, 3 Ghz processors, and 384 GB of memory. A time limit of one hour is imposed for all instances. The valid cuts are implemented using the CPLEX lazy constraint callback function.

A. COMPARING CCP WITH OTHER STOCHASTIC MODELS

We compare the performance of the proposed chance-constrained model with the two models proposed in [22]: SDORS-EV and SDORS-CVaR. SDORS-EV is a stochastic programming model that attempts to optimize the expected value of OR overtime and patient waiting costs. SDORS-CVaR is a risk-based model that minimizes the expected tail of overtime and waiting costs by using the CVaR function [22]. For simplicity, the following terminology is used in our experiments: CCP (chance-constrained), CVaR (SDORS-CVaR) and EV (SDORS-EV). The performance of these models are evaluated using several criteria.

Table 3 compares the performance of the three models after solving eight OR scheduling instances within the specified time limit. A finite set of 100 scenarios is generated for each surgery, and the parameter α_r is set to 0.10. The second and third columns show the number of surgical cases and available ORs for surgery operation. The column *Time* shows the computational time in seconds. Finally, the optimality gap reported in the last column is calculated as $\left(\frac{UB-LB}{UB}\right) \times 100\%$. It can be observed that the CCP model outperforms CVaR and EV in convergence speed. The chance-constrained model can solve all instances to optimality within the specified time limit while EV and CVaR only solve instances with up to 12 and 16 patients, respectively.

In Figure 1, the trade-off between minimizing costs and controlling the variability of costs is compared for each model. Several values are used for the confidence level parameter α in order to mimic the behavior of these models under different risk attitudes. A high α resembles an aggressive approach to minimize expected costs while accepting a substantial risk of OR overtime. On the contrary, a low α depicts conservative decision making (i.e., accepting higher costs given that the chance of OR overtime is low). As observed in Figure 1, CVaR places emphasis on minimizing variability while EV focuses on providing the minimum average costs. However, CCP provides a more moderate trade-off between minimizing average costs and reducing variability. Assuming a given tolerance for the OR schedule variability, CCP outperforms CVaR by providing more cost-effective solutions. Similarly, CCP outperforms EV by providing OR schedules with lower variability, assuming a fixed budget.

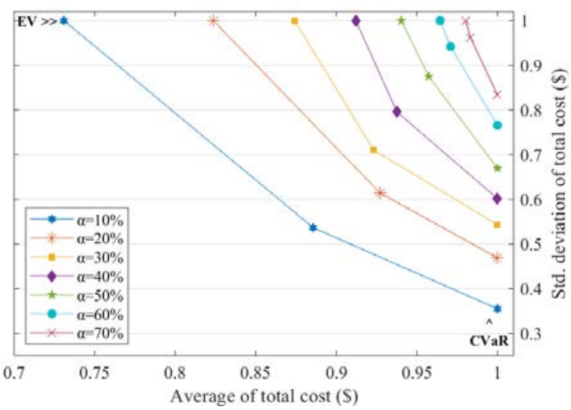

FIGURE 1: Trade-off between average total cost and variability of total cost

In Figure 2, we use several metrics to compare the performance of CCP, EV and CVaR under different values for α. The metrics are $[A, B, C, D, E]$ = [total cost, total waiting time & overtime, utilization, overtime scenarios, open ORs].

For small α values, CCP and CVaR suggest opening more ORs to reduce the risk of overtime and reduce the expected tail costs, respectively. Therefore, they incur higher average total costs and lower OR utilization than EV. EV displays better OR utilization at the risk of experiencing increased overtime. CPP is the superior method in terms of reducing overtime and patient waiting times. Moreover, CCP performs best in reducing the number of scenarios where overtime occurs. Overall, using CCP results in fewer occurrences of overtime and better OR utilization than CVaR when α is not very restrictive (i.e., $\alpha > 0.1$ in our numerical experiments).

TABLE 4: Performance of different feasibility cuts

Instance	Surgeries	ORs	**Cuts** (38)		**Cuts** (39)		**Cuts** (38) **&** (39)	
			Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)
1	6	5	2.5	0.0	1.9	0.0	3	0.0
2	7	5	2.6	0.0	2	0.0	5.1	0.0
3	9	5	8.2	0.0	7.5	0.0	9.8	0.0
4	12	5	69.1	0.0	75	0.0	49.6	0.0
5	16	5	205.6	0.0	243.7	0.0	131.3	0.0
6	20	5	216.2	0.0	285.2	0.0	150.5	0.0
7	23	10	357.9	0.0	405.1	0.0	231.7	0.0
8	29	10	653.4	0.0	732.9	0.0	405.4	0.0

TABLE 5: Performance of different solvers/algorithms on large-scale problems

Instance	Surgeries	ORs	Scenarios	CPLEX		Basic Decomposition		This Paper	
				Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)
9	40	10	100	3600	2.3	3600	1.5	713.8	0.0
	40	10	500	3600	5.6	3600	4.0	1400.3	0.0
10	63	20	100	3600	11.2	3600	9.8	1333.2	0.0
	63	20	500	3600	16.8	3600	13.4	2246.6	0.0
11	74	20	100	3600	19.4	3600	15.9	2270.8	0.0
	74	20	500	3600	28.7	3600	21.7	2631.6	0.0
12	89	20	100	3600	45	3600	32.3	2819.5	0.0
	89	20	500	-	-	3600	51.5	2851.2	0.0

It is also observed that the three models converge in all metrics as α increases.

Proposition 6. *CCP, CVaR and EV provide the same optimal solution when* $\alpha = 1$.

Proof. See Appendix A-H. □

B. SOLVING LARGE-SCALE TEST INSTANCES

It is observed in Table 3 that the solution times increase exponentially as the problem size grows. Therefore, we apply the valid inequalities and the decomposition algorithm presented in Section III to solve larger test instances in shorter time periods. We used the Monte Carlo sampling method to generate a set of 100 scenarios, and the parameter α_r was set to 0.10 Table 4 compares the performance of feasibility cuts (38) and (39) when used separately and combined to solve the test problem instances shown in Table 3. It can be observed that both valid inequalities are effective in reducing the solution time when compared to the cuts generated by the CPLEX solver. It is also observed that using both types of cuts leads to longer solution times for small test instances due to the time spent for generating inequalities. However, as problem size grows, adding both types of feasibility cuts to $\mathbb{F}$ leads to significantly faster convergence than applying them separately. Therefore, we use valid inequalities (38) and (39) to generate feasibility cuts in the following numerical experiments.

Larger test problem instances are solved and reported in Table 5 to evaluate the performance of the proposed decomposition algorithm. We compare the performance of our algorithm with that of the IBM CPLEX MIP Solver 12.9 [38] and a decomposition algorithm that uses the big-M optimality cuts introduced in [17]. The solution time and the optimality gap are reported for each algorithm. The column *Basic Decomposition* illustrates the results from the decomposition algorithm using feasibility cuts (38) and big-M optimality cuts. The last column highlights the results of the proposed decomposition algorithm in this paper. As shown in Table 5, we observe that the CPLEX solver is the least desirable option for solving M_{DEF}, as expected. For the largest problem instance, the CPLEX solver does not find any feasible solutions within the time limit. User-defined feasibility and optimality cuts can improve solution speed. It can also be observed that using stronger optimality cuts rather than the big-M cuts can reduce solution time significantly. Neither the CPLEX solver nor the basic decomposition algorithm can solve any of the instances to global optimality within the time limit. Nevertheless, the proposed decomposition algorithm outperforms other methods by solving all test instances to optimality within 48 minutes.

C. IMPORTANCE OF MINIMIZING EXPECTED COSTS

Our numerical experiments show that a significant portion of total costs comes from the expected overtime and waiting time costs. Neglecting these measures in OR scheduling models can result in surpassing the predicted overtime budget by 200%, disheartening staff from longer-than-expected shifts, and causing dissatisfaction to patients [8]. We solved 10 replications of all test instances in Table 3 using two different objective functions: objective (21); and objective

(21) minus the expected second-stage costs. Then, we calculated the sum of costs pertaining to OR opening, OR overtime, and patient waiting time for each optimal solution of each case. The percentage of savings obtained from optimizing both deterministic and stochastic costs is calculated as $\left(\frac{obj_2^* - obj_1^*}{obj_2^*} \times 100\%\right)$. It is observed from Figure 3 that minimizing the expected costs leads to greater savings when larger α values are used. This highlights the importance of minimizing the expected costs in addition to satisfying the chance constraints when solving stochastic OR scheduling problems.

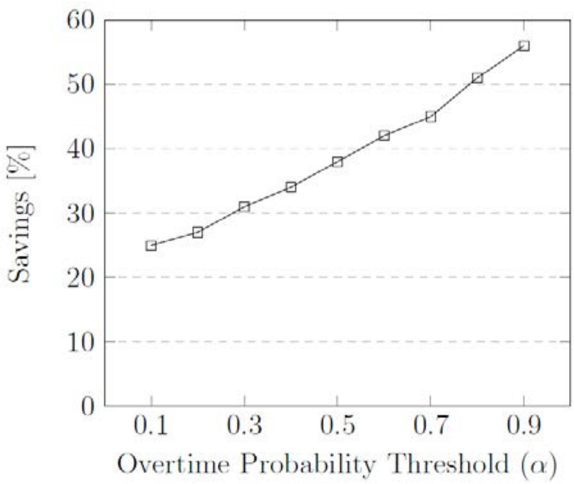

FIGURE 3: Advantage of minimizing expected costs when solving the chance-constrained OR scheduling model

D. INDIVIDUAL VS. JOINT CHANCE CONSTRAINTS

The chance constraints in Section II-B are enforced on each OR independently. However, a decision-maker might be interested in controlling the chance of OR overtime on an aggregate level. In such cases, the chance constraints (12) are replaced by:

$$Pr\{tr_{krs} + \sum_{i \in I} \delta_{is} y_{ikr} \leq cap_r : \forall k, r, s\} \geq 1 - \alpha \quad (51)$$

This section compares the joint chance-constrained OR scheduling model (M_{Joint}) with the proposed model M_{DEF} presented in Section II. In the following numerical experiments, the Monte Carlo sampling method is used to generate a set of 100 scenarios for each test instance. We applied the decomposition algorithm proposed in Section III and the same classes of valid inequalities to both models. Figures 4 and 5 compare M_{DEF} and M_{Joint} after solving the larger test instances shown in Table 5. Figure 4 illustrates that M_{Joint} tends to open more ORs to satisfy the tighter limit on OR overtime. The joint chance constraints restrict the occurrence of overtime to $\alpha|\mathbb{S}|$ scenarios while the individual chance constraints allow up to $\min\{\alpha_r|\mathbb{S}||R|, |\mathbb{S}|\}$ scenarios with OR overtime. As the probability threshold α loosens, the gap between the optimal number of open ORs obtained by the two models shrinks due to the converging feasible regions. Similar to Proposition 6, it can be shown that the two models achieve equivalent optimal solutions when $\alpha = 1$. Figure 5 compares the performance of M_{Joint} and M_{DEF} in reducing the OR overtime and patient waiting times. The vertical axes depict the average overtime and waiting time per OR per scenario, respectively. It can be observed that M_{Joint} achieves greater success at controlling OR overtime by opening more ORs and setting the projected start times earlier to satisfy the stricter chance constraints. However, these measures lead to lower OR utilization and higher waiting times when compared to that of M_{DEF}.

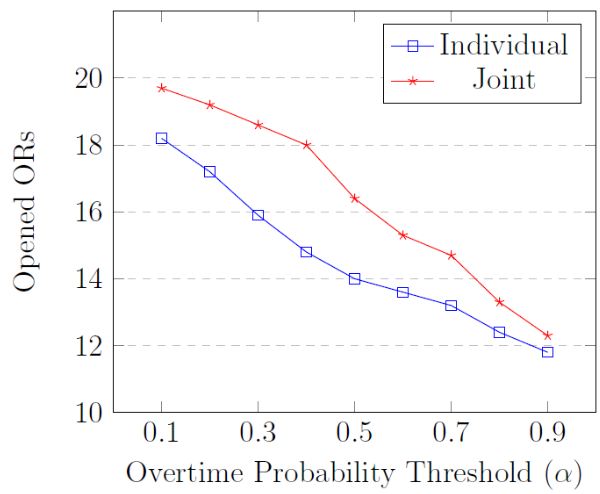

FIGURE 4: Optimal number of open ORs

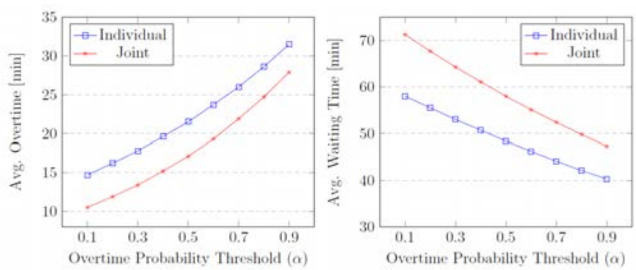

FIGURE 5: Optimal overtime and waiting times

Table 6 compares the run times of the two models after solving the test instances presented in Table 3 and Table 5. Column *First Feasible* shows the run time to find the first feasible solution. Column *1% Gap* details the run time to reach 1% optimality gap. It is observed that M_{DEF} can find feasible solutions and attain near-optimal solutions in shorter time lengths compared to M_{Joint}. For the largest test instance, M_{DEF} finds a feasible solution within half the time required by M_{Joint} and solves the problem to optimality within 48 minutes. On the other hand, M_{Joint} fails to reach 1% optimality gap within the one-hour time limit.

TABLE 6: Computational performance of M_{Joint} and M_{DEF}

Instance	M_{Joint}		M_{DEF}	
	First Feasible (s)	1% Gap (s)	First Feasible (s)	1% Gap (s)
1	1.2	4.7	0.8	3.1
2	1.7	7.8	1.3	5.5
3	2.6	13.8	1.8	9.5
4	7	50.1	5	38.8
5	26.6	151	19.7	111.1
6	23.5	163.7	16.6	121.9
7	50.7	244.4	34.8	179.3
8	73.8	448.3	52.7	317.4
9	137.1	988.9	80.2	649.3
10	296.8	2066	172.5	1312.2
11	519.1	3056.2	339.3	2090.7
12	616.4	3609.9	316.8	2594.6

V. CONCLUSIONS

In this paper, a two-stage chance-constrained mixed-integer programming model was proposed for the OR scheduling problem with stochastic surgery durations. The individual chance constraints controlled the risk of OR overtime. The goal was to minimize the sum of OR opening, OR overtime and patient waiting costs. Our model was compared with two other stochastic models in the literature: an expected value model and a CVaR-based model. We used several criteria such as computational efficiency, mean-variance trade-off for the total costs, and OR utilization. We demonstrated that minimizing the expected costs when solving the chance-constrained OR scheduling model results in significant savings compared to the case where only the deterministic costs are minimized. Moreover, we compared the individual and joint chance constraints in terms of allocated ORs, second-stage stochastic costs and solution times. A decomposition algorithm with strong feasibility and optimality cuts was applied to effectively solve large-scale test instances. We proposed an algorithm that generated feasibility cuts using the first stage solutions, and as a result, significantly reduced the time required to find feasible solutions. Numerical experiments demonstrated that the decomposition algorithm outperformed both the IBM CPLEX solver and a basic decomposition algorithm by solving the largest test instances to optimality within the one-hour time limit. Moreover, it is shown that the individual chance constraints lead to higher OR utilization, reduced patient waiting times and shorter solution times. It is demonstrated that finding strong cuts can increase the convergence speed significantly. Therefore, discovering stronger feasibility and optimality cuts in order to solve larger problems in shorter time frames can be a promising topic for future research.

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APPENDIX A PROOFS

A. PROPOSITION 1

Proof. According to constraints (12) and (13), an overtime occurs if:

$$tr_{krs} + \sum_{i} \delta_{is} y_{ikr} > cap_r, \forall k \in K, r \in R, s \in \mathbb{S} \quad (52)$$

Therefore, the value of z_{rs} in (16) captures scenarios where an OR runs overtime. Given that the random surgery duration has a discrete and finite support, constraint (17) limits the number of scenarios where each OR can run overtime. □

B. ASSUMPTION 3

Proof. Since all of the first-stage decision variables are binary, $\mathbb{P}(s)$ is bounded by a hypercube of dimension N. Therefore, $\theta = 0$ is the only solution that satisfies the condition in the definition of C. In other words, $C = \{0\}$ for all $s \in \mathbb{S}$. □

C. ASSUMPTION 4

Proof. We need to show that both first-stage and second-stage problems have bounded optimal solutions. We know from Assumption 2 that both problems are feasible. The highest objective function value for the first-stage problem is when all operating rooms are open, i.e., $\sum_r f_r$, which is bounded. Given the first-stage solution, every open operating room will run overtime in up to $\alpha_r|\mathbb{S}|$ scenarios. The amount of overtime is bounded by $\max_{k\in K}\{tr_{krs} + \sum_i \delta_{is} y_{ikr} - cap_r\}$, which is also a finite value given $tr_{krs} \geq 0$ and the minimization objective function in the second-stage problem. □

D. PROPOSITION 2

Proof. This holds for the individual chance constraints in our model without loss of generality. Assume that in the optimal solution to our model, there exists $r \in R$ where $\sum_s z^*_{rs} = \epsilon < \alpha_r|\mathbb{S}|$. The optimal solution will allow

$(\alpha_r|\mathbb{S}| - \epsilon)$ scenarios to run in overtime mode ($z_{rs} = 1$) with the corresponding overtime variables o_{rs} set to zero. □

E. PROPOSITION 3

Proof. First, we know from Assumption 2 that $\mathbb{P}(s) \cap X \neq \Phi$. Given that $\bar{X} \supseteq X$, we have $\mathbb{P}(s) \cap \bar{X} \neq \Phi$ that ensures the feasibility of (30). Second, the single-scenario optimization subproblem is bounded if γ is chosen to be a vector in the dual cone of C (recession cone). The dual cone of C is defined as $C^* = \{\gamma \in \mathbb{R}^N \mid \gamma\theta \geq 0, \forall \theta \in C\}$. From Assumption 3, we know that $C = \{0\}$ for model (M_1). Therefore, any $\gamma \in \mathbb{R}^N$ is a vector in C^*. □

F. PROPOSITION 4

Proof. From Assumption 4, we know that both (18) and (19) are non-negative and bounded. We also know that x is binary. It suffices to have $\tau \in \mathbb{R}_+^N$ such that $\psi_{rs}^1(\tau) > -\infty$ and $\psi_{rs}^2(\tau) > -\infty$. Therefore, $D = \mathbb{R}_+^N$ satisfies the condition. □

G. THEOREM 2

Proof. The feasibility cuts are added to the master problem (29) to remove the first-stage decisions that result in infeasible M_2. It is known from Assumption 2 that the set of feasible solutions to M_1 is finite. Therefore, a finite number of inequalities of type (38) and (39) can be added to (29). Moreover, the sets Π_{rs} and $\bar{\Pi}_{rs}$ of the optimal solutions to the dual problems in Section III-B are finite since there is no constraint parallel to the objective function (40). Therefore, a finite number of optimality cuts (50) will be generated. Given that there are finite numbers of feasibility and optimality cuts, Algorithm 2 converges in a finite time following the convergence of the Benders decomposition algorithm [26]. □

H. PROPOSITION 6

Proof. When $\alpha = 1$, the chance constraints (12) can be written as:

$$Pr\{tr_{krs} + \sum_i \delta_{is} y_{ikr} \leq cap_r : \forall k, s\} \geq 0, \forall r \in R \quad (53)$$

which holds for all feasible solutions to the first-stage problem. Therefore, the chance constraints are redundant and we have $CCP \equiv EV$ when $\alpha = 1$. Now, it suffices to show that $CVaR \equiv EV$. CVaR is defined as the expectation of those outcomes where total costs exceed a threshold value, called Value-at-Risk (VaR) [27]. For $\alpha = 1$, the VaR of second-stage costs is defined as:

$$VaR_1 = \min\{g(x,s) : CDF(g(x,s)) \geq 0\} \quad (54)$$

where CDF represents the cumulative density function. Given that $CDF \geq 0$ for every random variable, we conclude:

$$g(x,s) \geq VaR_1, \forall x \in X, s \in \mathbb{S} \quad (55)$$

Therefore, from Assumption 2 and inequality (55), the CVaR model minimizes the total costs over all scenarios, thus indicating equivalence to using the EV model. □

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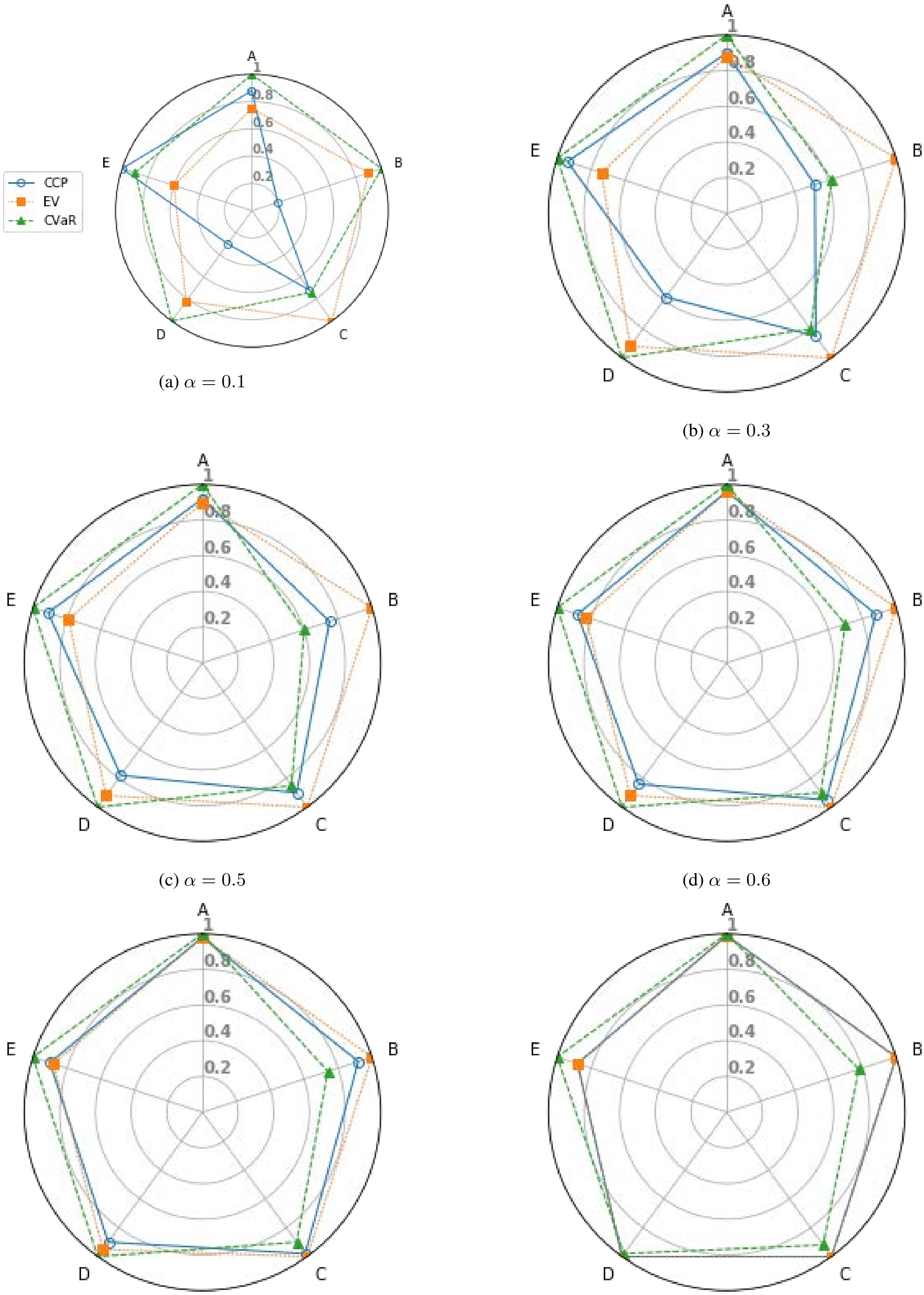

(a) $\alpha = 0.1$

(b) $\alpha = 0.3$

(c) $\alpha = 0.5$

(d) $\alpha = 0.6$

(e) $\alpha = 0.7$

(f) $\alpha = 0.8$

FIGURE 2: Impact of using different risk thresholds on the performance of CCP, EV and CVaR