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A Robust Chance Constraint Programming Approach for Evacuation Planning under Uncertain Demand Distribution

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Abstract

We consider the evacuation planning problem where the number of actual evacuees (demand) is unknown at the planning phase. In the context of mass evacuation, we assume that only partial information of the demand distribution (i.e., moment, support, or symmetry) is known as opposed to the exact distribution in a stochastic environment. To address this issue, robust approximations of chance-constrained problems are explored to model traffic demand uncertainty in evacuation networks. Specifically, a distributionally robust chance-constrained model is proposed to ensure a reliable evacuation plan (start time, path selection, and flow assignment) in which the vehicle demand constraints are satisfied for any probability distribution consistent with the known properties of the underlying unknown evacuation demand. Using a path based model, the minimum clearance time is found for the evacuation problem under partial information of the random demand. Numerical experiments show that the proposed approach works well in terms of solution feasibility and robustness as compared to the solution provided by a chance constrained programming model under the assumption that the demand distribution follows a known probability distribution.

Keywords: Short notice evacuation, dynamic network flow problem, partial information, distributionally robust chance-constraint.

1 Introduction

Disaster events such as hurricanes and flooding can result in loss of lives as well as massive property damage, which can be minimized by efficient distribution of resources for both the inbound and outbound logistics (Abdelgawad et al. 2009; Renne et al. 2011). Outbound evacuation logistics is highly uncertain and the traffic demand is not known in advance. The rate of participation of would-be evacuees during an emergency evacuation depends on a number of factors including the nature of the disaster (e.g., natural/man-made), the type of dwellings involved (e.g., permanent versus mobile homes), the region of impact, the time of impact, and the evacuees' perception of their risk. In order to mitigate the undue consequences arising from uncertain demand, we study the problem of generating evacuation transportation plans that are robust to random outgoing demand, i.e., unknown number of evacuees at each source node such as a ZIP code.

Large-scale evacuations are rare events and because of their uniqueness, limited data regarding them is available. Demand estimates are usually based on expert judgment, which can lead to difficulties in forming a reliable estimate of the associated demand distribution, creating

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inconsistencies in estimation. In some contexts, limited historical data may suffice to roughly estimate the mean demand, which allows us to formulate and solve the so-called expected-value problem (Ahmadian et al. 2016; Veismoradi et al. 2016; Gao et al. 2018), in which the uncertain parameters take their mean values. However, in the context of evacuation planning when some unplanned event occurs, an expected-value plan may seriously impair the effectiveness of such an evacuation plan. This suggests that we should address uncertain input parameters by explicitly accounting for the uncertainty, aiming for a robust plan that limits the negative impact of requisite real-time recourse actions. In the optimization context, a "worst-case" demand scenario is typically assumed (Wolshon, 2009) and, again, a single-scenario evacuation problem is solved, leading to a long optimum clearance time (the soonest time at which all evacuees have been evacuated). In our view, we should carefully balance the operational failure probability due to high evacuation demand and optimum clearance time by allocating available capacity.

One way to account for demand uncertainty is to formulate the problem using a chance-constrained program of the following form:

$$\min\ f(x) \tag{1a}$$

$$\text{s.t. } \mathbb{P}\left(F\left(x.\tilde{\xi}\right) \leq 0\right) \geq 1-\epsilon, \tag{1b}$$

Where $\tilde{\xi}$ is a random vector with an associated probability measure $\mathbb{P}$, F is a function describing a performance measure in a particular system, and f is the objective function. In an evacuation planning problem, $\tilde{\xi}$ describes the random vector of demand for evacuees from multiple source nodes, function F is the difference between the outgoing traffic flow and the random demand from a source node, and f is the number of evacuees left behind. We fix a probability level $\epsilon \in [0,1]$ that requires the constraints of the system to be satisfied with a confidence level of at least $(1-\epsilon)$.

The way we tackled the problem from an algorithmic perspective strongly depends on what we know about the probability distribution of the uncertain parameters and the form of F. An assumption made in the chance-constrained model above is that the probability distribution of the underlying random parameter is known. However, in many cases, it may be impossible to accurately estimate this distribution. Due to insufficient data and differences in the nature of disasters, accurate estimation is particularly difficult when considering an evacuation problem. One may replace the unknown distribution $\mathbb{P}$ in (1b) by a crude estimate, like $\widetilde{\mathbb{P}}$. However, such an approach may lead to an overly optimistic solution, which fails to satisfy the chance constraint under the "true" distribution, $\mathbb{P}$. If we want to evaluate, bound, or approximate the probability in the chance constraint, we have to make some assumptions about the probability distribution. A more realistic assumption may be that we have limited information regarding $\mathbb{P}$; e.g., information about its moments or support. These assumptions consequently affect the optimal choice of evacuation routes and traffic flow in an evacuation plan.

In this work, we use evacuation planning model of Lim et al. (2012) because it can handle very large-size evacuation networks and we incorporate uncertainty of demand in the optimization model. Using the distributionally robust chance-constrained modeling concept, we reformulate the model into the following form:

$$\min\ f(x) \tag{2a}$$

$$\text{s.t. } \mathbb{P}\big(F(x.\tilde{\xi}) \le 0\big) \ge 1-\epsilon,\ \ \forall \mathbb{P} \in \mathcal{P}. \tag{2b}$$

In the distributionally robust setting, the probabilistic constraint is satisfied for the set of all possible probability distributions in $\mathcal{P}$ that are consistent with the known properties of $\mathbb{P}$, such as its first and second moments or its support. Following assumptions about the probability distribution of the underlying demand are made: (i) the cumulative distribution function (CDF) for the demand is unknown but partial moment information, such as the first and second moments are known; (ii) the demand distribution is symmetric (Groen and Polivka 2008, Yin and Gladwin 2014); (iii) the range of the demand is known (Groen and Polivka 2008). From practical point of view, information for assumptions (i) and (iii) can be obtained for a mass evacuation as opposed to the exact distribution for the number of people who will be evacuating from a given location. Solutions that are robust with respect to variations of demand within the specified class of distributions can help evacuation managers to assess the quality and reliability of an evacuation plan according to the number of evacuees left behind prior to the implementation of the evacuation plan. However, such a plan needs to be generated in a timely manner. Hence, one of the main contributions of this paper is to develop computationally tractable approximations of distributionally robust chance-constrained programs corresponding to demand uncertainty.

2 Literature review

In the literature of evacuation planning oftentimes network flow optimization concept is employed in the proposed approaches (Lu et al. 2005; Kim et al. 2008; Bretschneider and Kimms 2011). Comprehensive survey on evacuation planning can be found in Hamacher and Tjandra (2002), Wolshon et al. (2005), Yusoff et al. (2008), Abdelgawad et al. (2009), Renne et al. (2011), Murray-Tuite and Wolshon (2013), and Bayram et al. (2016). The vast body of the literature has focused on deterministic evacuation plans (Church and Cova, 2000; Bish et al. 2014; Gan et al. 2016; Pillac et al. 2016; Qazi et al. 2017; Hu et al. 2017; Aalami and Kattan 2018; Renne 2018; Bolia 2018; Gai et al. 2018; Li et al. 2018; etc). However, assuming deterministic demand can lead to poor results due to inability of solutions to deal with deviations of demand. As found in (Lindell and Prater 2007), there was a large difference between the estimated number of evacuees (686,000) and the actual number of evacuees (1,800,000) from the Greater Houston area during Hurricane Rita, which led to dramatic traffic congestion and delay.

A number of demand-loading models have been proposed for use in evacuation planning, which is associated with time dependent demand. Sherali et al. (1991) studied the location allocation problem in evacuation networks considering a constant demand dissipation rate over time. Yazici and Ozbay (2010, 2008) found that variations in the demand curve over time and variations in the level of the demand have a significant effect on the optimum clearance time. The authors further propose a probabilistic model for evacuation planning considering demand and capacity uncertainty. Mainly demand-loading representation includes the so-called S-curve (Hobeika and Jamei 1985; Radwan et al. 2005), Rayleigh distribution (Tweedie et al. 1986), and sequential logit model (Fu and Wilmot 2004). There also exists studies that consider time independent demand. Chiu et al. (2007) assumed that all demand for each origin destination (O-D) pair of the network is loaded on each corresponding source node at once and is independent from time. This paper also assumes that the estimated number of evacuees in a specific area (i.e., total projected demand of the source node) will be present at the beginning of the planning horizon. Although the actual number of evacuees for an upcoming event is not known in advance, partial information about intended evacuees can be captured using the census data of the region.

Research focusing on uncertainty of demand and capacity in evacuation modeling has been addressed via chance-constrained programming and robust optimization. Waller and Ziliaskopoulos (2006) first used a chance-constrained program for the traffic assignment problem under a uniform distribution for traffic demand. Ukkusuri and Waller (2008) propose a two-stage stochastic program with recourse to account for demand uncertainty. They show that not accounting for demand uncertainty can significantly degrade the quality of the resulting solution. Wang et al. (2016) use scenario-based stochastic program to deal with uncertain capacities and travel times. Three criteria is considered for evaluating traffic routing plans and crisp linear equivalents of the strategies is used in the solution methodology. Yao et al. (2009) apply robust optimization technique to address demand uncertainty. Yao et al. (2010) studied provided an affinely Adjustable Robust Counterpart (AARC)) based linear programming model using concept of Cell Transmission Model considering demand uncertainty sets as box or polyhedral sets. Chung et al. (2011) use box uncertainty sets to provide a linear tractable robust model for a system optimal dynamic traffic assignment model (SO DTA) under demand uncertainty. Goergik, et al. (2016) consider solution ranking as well as objective ranking robustness for the problem of evacuation planning. In their approach, degree of robustness of a solution is defined by using solution ranking procedures which include both quantitative and qualitative aspects. Bish and Sherali (2013) examine the effectiveness of aggregate-level and staged evacuation process as a demand-based strategy in an evacuation network under demand uncertainty. The CTM based model is used to compare effects of free-flow strategies with strategies under congestion in relation to tractability, normative optimality, and robustness of the solution. Pourrahmani et al. (2015) assumed demand on source nodes of an evacuation network to be as a fuzzy number, and provided a genetic algorithm based on fuzzy credibility theory to solve the optimization model. Goerigk et al. (2015) proposed a two-stage bi-criteria robust formulation for evacuation using buses considering vulnerability of the schedule to varying evacuation circumstances. Swamy et al. (2016) have used a simulation tool to depict stochastic arrival of demand, evacuees dispatch, and queueing effects at the pickup locations.

Tarhini and Bish (2016) used a cell transmission model (CTM) for a dynamic traffic assignment (DTA) problem considering system optimal (SO) approach instead of commonly deployed user-equilibrium based concept. This makes the model to be relevant to regional evacuation planning problems. Kimms and Maiwald (2018) have proposed a path-based evacuation model based on the concept of CTM model having two objective function and taking into account uncertainty in roads of the network.

Yazici and Kaan (2007) proposed a chance constraint program to address uncertainties in road capacities when distribution of the capacity of the links are known. Lim et al. (2015) use a chance-constrained model to analyze the reliability of an evacuation plan considering the uncertain capacity of road links where the uncertain capacity is modeled using a Weibull distribution. Lv et al. (2013) applied a joint-probabilistic constrained (JPC) technique for the case of nuclear emergency evacuation. In the proposed model, uncertainties expressed as joint probability and interval values are addressed by incorporating interval-parameter programming and joint-chance constrained techniques. In all of the above studies, a priori knowledge of the underlying distribution is required.

To account for unknown distribution, Ng and Waller (2010) and Ng et al. (2011) derive bounds on travel time reliability. The bound proposed in their approach is based on Markov's inequality. However, the proposed bound may be loose in the context that further information about the underlying distribution is known. Ben-Tal et al. (2009) provided a robust linear programming model for the case of surface transportation networks assuming uncertainty in data which penalizes loss of life or property. Ben-Tal et al. (2011) extend a robust optimization approach for multi-period transportation problems and apply an affinely adjustable robust counterpart (AARC) approach to consider "wait and see" decisions for dynamic traffic assignments. They apply the robust optimization framework to an emergency logistics planning problem and show that the AARC solution provides excellent results when compared to the solutions from deterministic linear programming and stochastic programming based on Monte Carlo sampling. Lv et al. (2015) couple a chance-constrained programming with an interval chance-constrained integer program (EICI) in order to cope with interval uncertainties that cannot be addressed by any specific distribution functions. Chung et al. (2012) use moment information to formulate a distributionally robust chance-constrained model which allows them to derive a deterministic approximation of their model. Ng and Lin (2015) proposed an approximation of chance-constrained cell transmission model (CTM) for the case that only the first and second moments of demand and capacity are known. Although the probability inequalities that they used for demand constraint is similar to the work of Ng and Waller (2010), by using Cantelli's inequality, they٩ provide sharper equalities for approximating capacity constraints.

In our work, we provide a more comprehensive approach to address demand uncertainty not only for the case that mean and variance of demand distribution are known, but also for the case that additional information such as the demand uncertainty with symmetry and/or support information are available. Using this additional information, we prove that tighter bounds can be achieved. The transportation network is presented by a directed graph of nodes and arcs and e a robust chance constraint path-based evacuation plan is provided that compared to cell transmission models (CTM) can be efficient in salving large-scale evacuation networks. Our approach is rooted in assessing the number of people left behind at the affected area using an evacuation plan within a given evacuation time. This approximation method provides a better estimate of the evacuees' demand based on further information on the distribution, as compared to using just the moment information and making the demand estimate.

General chance-constrained models are computationally intractable. Significant research efforts are focusing on coming up with safe and tractable approximation of chance constraints (Geletu et al. 2013; Zaghian et al. 2017; Hamian et al. 2018). Chance-constrained models without full knowledge of underlying distribution have recently been solved using various approximation approaches. One such approach is proposed by Calafiore and Ghaoui (2006). They show that a chance constraint is second-order cone representable based on moment, support or symmetric information of the uncertainty. More generally, they show that for $\epsilon \leq 0.5$ individual chance constraints can be converted to second-order cone constraints whenever the random vector $\tilde{\xi}$ is governed by a radial distribution. We use a similar approach to derive a deterministic approximation of the chance constraint for various approximations of the demand distribution.

Nemirovski and Shapiro (2007) develop a Bernstein approximation for chance-constrained models that are convex and efficiently solvable. Ben-Tal et al. (2010) propose a soft robust optimization framework that relaxes the standard notion of robustness by allowing the decision maker to vary the protection level in a smooth way across the uncertainty set. Recently, Calafiore and Campi (2005), Erdogan and Iyengar (2006) and Luedtke and Ahmed (2008) have proposed to replace the chance constraint (1b) by a point-wise constraint that must hold at a finite number of sample points drawn randomly from the distribution$\mathbb{P}$. The advantage of this Monte Carlo approach is that no structural assumptions about $\mathbb{P}$ are needed and the resulting approximate problem is convex. However, the drawback of such sampling based methods is that they may be too computationally intensive to solve large problems or to solve problems for which a small violation probability ϵ is required. Zymler, et al. (2013) developed tractable semi-definite programming base approximations for both individual and joint chance constraints for the case that second-order information is available. This approximation can only be achieved when the decision vector is a convex closed set.

Cell transmission models (CTMs) are commonly used in the evacuation literature because of their capability to capture the traffic dynamics (Chung et al. 2011; Bish and Sherali 2013; Bish et al. 2014; Tarhini and Bish 2016). It does well in presenting flow propagation and modeling reduced travel times from congestion. The original CTM model (Daganzo 1994, 1995) is computationally expensive because of its nonlinear flow-density relationship (Peeta and Ziliaskopoulos 2001). To overcome this drawback, Ziliaskopoulos (2000) introduced a linear version of CTM which has been widely cited in the literature. However, we adopted a path-based model (PBM) introduced by Lim et al. (2012). PBM is much closer to the evacuation planning process that the authors are involved with at a local level, and it can handle very large-scale evacuations networks. The PBM also prevents traffic hold-back on the roads that can happen in the LP CTM because, in theory, a CTM-based model allows vehicles to be stored at a cell.

Our developed plan by the PBM intends to minimize the number of people left behind at evacuation nodes within a given evacuation horizon when we have scenario of demand uncertainty. Contribution of our work is in proposing robust approximation of the uncertain demand constraint when probability distribution function of demand is unknown and only partial (Moments, Symmetry, and Support) information is available.

The rest of the paper is organized as follows: in Section 3, we describe the network flow optimization problem in a transportation network for evacuation routing and scheduling. In Section 4, we introduce a robust approximation method for handling our chance constraint along with the corresponding models. The applicability of the model is demonstrated with a sample network in Section 5, as well as our results for various scenarios that might occur during evacuation. Conclusions and future work are presented in Section 6.

3 Problem formulation

A dynamic network flow model has been used to mathematically represent traffic flow evolution in an evacuation network. A dynamic network can be visualized as a static network with an additional dimension representing time, i.e., the static network is repeated for each discrete slice of time (Aronson 1989; Ford and Fulkerson 1958, 2015; Xuan et al. 2003). Traffic assignment on such time-expanded networks relies upon a more aggregate representation of traffic as a series of flows that attempts to match the demand for road space with the capacity of the highway system's links and intersections at various time.

Let us consider a directed network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ consisting of a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$. For each arc $a \in \mathcal{A}$, we define θ_{pa} as the transit time on arc a of path p and C_a as the arc capacity. Nodes in the network are categorized into source nodes ($\mathcal{N}_s$), intermediate nodes, and destination nodes ($\mathcal{N}_d$). Let S_i be the number of evacuees at source node $i \in \mathcal{N}_s$ and ℓ_j be the capacity of destination node $j \in \mathcal{N}_d$. We assume that there are T time periods $\mathbb{T} = \{0, 1, \ldots, T-1\}$ to complete the transportation of evacuees from source nodes to the destination nodes.

3.1 Path Based Model

In this section, we present a deterministic evacuation route planning model. To formulate such a problem, much of the work in the literature uses a network flow optimization model that finds the flow on roads with respect to limited capacities of the roads. In short-notice emergency evacuations, egress paths are decided a priori by the authorities and the main issue is traffic control and flow management to have a smooth evacuation. For large evacuation networks, finding the evacuation paths, corresponding flow and schedule is computationally intractable and various heuristic methods are applied to find an evacuation plan (Bretschneider and Kimms 2011; Kim et al. 2008; Lim et al. 2012; Al Qhtani et al. 2017).

Here, we use a path-based model (PBM) because it is scalable for large evacuation networks, and it is easy to implement in practice. In PBM, a set of evacuation paths is first selected from a pool of all possible paths between the source and destination. Paths can be enumerated applying successive shortest-path algorithm or using CPLEX solution pool for shortest-path problem. By feeding candidate paths into the PBM, flow and schedule of the paths are determined for each origin-destination (O-D) pair. Hence, this approach reduces the problem complexity by enumerating possible paths to be included in the model instead of leaving the selection of paths to the optimization model as is the case for an arc-based evacuation network model. Rungta et al. (2012) justified the use of PBM using an evacuation network (see Figure 3) in which the PBM found an optimal solution in few seconds, while the arc-based model could not find a feasible solution after three hours of computation. The PBM also has the ability to address specific desirable functions such as limiting the number of used paths in the plan and eliminating paths with high durations.

In the context of evacuation planning two approaches can be used for flow and path assignment. One approach is to use the paths that have been formed by the evacuation planning optimization models which simultaneously develop flow schedule. For instance, in arc-based evacuation models in which paths are formed and selected at the same time that the flow schedule is provided. However, while working with evacuation mangers in a metropolitan area, we have come across another approach that can be used in practice. Emergency managers are mostly used to work with few limited path options in which they would find more familiar. Hence, the approach that we employ is to first enumerate many alternative paths in advance (e.g. using solution pool of shortest path problem) and improve them based on planners' experience and/or personal preference, and then use a similar model to our PBM model to find traffic assignments including evacuation schedule. Benefit of using a path pool based optimization is that it can help managers to get exposed to a better selection of evacuation paths that were not considered by them in their previous practice for the region. Using path-based models, one must ensure that the feasible path pool to the model should be large enough to contain good near-optimal solutions.

Assuming a limited number of evacuees (demand) at source nodes and a limited capacity at destination nodes, the objective function of DPBM is to evacuate the maximum number of people from the evacuation zone within a given planning horizon T. The model is designed to select a set of evacuation paths and assign corresponding flow rates such that the objective minimizes the total remaining evacuees at the end of time horizon. Evacuees left behind at the source node at the end of time horizon are represented as a penalty variable for unserved demand and their summation over all the source nodes is minimized. This ensures that the total number of evacuated people is maximized within time T. We can also apply Algorithm 1 introduced by Rungta et al. (2012) on the DPBM to minimize CT and calculate the earliest time at which the network can be cleared. Note that the objective function of DPBM is to minimize $\sum_{i \in \mathcal{N}_s} \beta_i$ within a given planning horizon T. It turns out that the minimum CT is achieved when we find a minimum planning horizon T in which objective function of a PBM (here DPBM) is equal to zero ($\sum_{i \in \mathcal{N}_s} \beta_i = 0$). In Algorithm 1 in Rungta et al. (2012), planning horizon T is initialized with a lower bound and is given as an input to the PBM model. If $\sum_{i\in\mathcal{N}_s}\beta_i = 0$, selected paths and assigned flow and schedule on those paths can clear the network in T, i.e., $CT^* = CT$. Otherwise, if the $\sum_{i\in\mathcal{N}_s}\beta_i \neq 0$, the planning horizon T is increased by one unit. The process is repeated until CT^* is obtained.

We denote the set of paths by $\wp$, the set of paths with source node i by $\wp_i^+$, and the set of paths with destination node j by $\wp_j^-$. There are two sets of integer decision variables in the model:

$f_{pt} \in \mathbb{Z}^+$: Flow on path $p \in \wp$ at time t

$\beta_i \in \mathbb{Z}^+$: Unsatisfied demand associated with source node $i \in \mathcal{N}_s$.

Using the notation in Table 1, the deterministic path-based model (DPBM) can be presented as follows:

Table 1: Notation

Notation	Description
$\mathcal{N}$	Set of all nodes
$\mathcal{N}_s$	Set of all source nodes
$\mathcal{N}_d$	Set of all destination nodes
C_a	Capacity of arc a
S_i	Demand of source node i
τ_a	Arc a travel time
ℓ_j	Capacity of destination node j
$\wp_i^+$	Set of paths originating from source node i
$\wp_j^-$	Set of paths terminated at destination node j
$\wp$	Set of all paths

Minimize
$$\sum_{i\in\mathcal{N}_s}\beta_i \qquad \textit{(DPBM)} \tag{3a}$$

Subject to:
$$\sum_{p\in\wp_i^+}\sum_{t\in\mathbb{T}} f_{pt} + \beta_i \geq S_i, \qquad \forall i \in \mathcal{N}_s, \tag{3b}$$

$$\sum_{p\in\wp}\delta_{pa} f_{p(t-\theta_{pa})} \leq C_a, \qquad \forall a \in \mathcal{A},\ \forall t \in \mathbb{T}, \tag{3c}$$

$$\sum_{p\in\wp_j^-}\sum_{t\in\mathbb{T}} f_{pt} \leq \ell_j, \qquad \forall j \in \mathcal{N}_d, \tag{3d}$$

$$f_{pt} \in \mathbb{Z}^+ \qquad \forall p \in \wp, \forall t \in \mathbb{T}, \tag{3e}$$

$$\beta_i \in \mathbb{Z}^+ \qquad \forall i \in \mathcal{N}_s \tag{3f}$$

Constraint (3c) limits the total flow on each arc to the capacity of the arc. In this constraint, parameter δ_{pa} is a binary parameter which takes value 1 if path p contains arc a and 0 otherwise. Variable $f_{p(t-\theta_{pa})}$ ensures that the flow originating on path p at time $t-\theta_{pa}$ reaches arc a after the transit time θ_{pa}. This constraint allows the simultaneous sharing of any arc by multiple paths. Constraint (3b) is related to demand at each source node and guarantees that the sum of flows on paths originating from each origin node in $\mathcal{N}_s$ over all time and the number of evacuees left behind represented by penalty variable β_i is equal to initial demand at that node. However, we convert it to an inequality constraint as seen in (3b) because the inequality constraints improved the computational convergence of the model. Constraint (3b) together with the objective function (3a) minimize the summation of unsatisfied demand β_i from all source nodes. This objective function, in turn, maximizes the total outgoing flow from the network. Constraint (3d) bounds the total incoming flows at each destination node to its capacity. Constraints (3e) and (3f) reflect the non-negativity and integrality conditions. The model could have been written in other ways to reduce the number of variables but the current model is a natural way to address the shortfall in the number of evacuees left behind within a given planning horizon T.

For the DPBM model, a deterministic estimate of the demand is used on the right-hand side of constraint (3b). The evacuation plan based on the assumption of deterministic demand may result in an optimal value which may not reflect the true number of people left behind when the actual demand differs from its estimated value. Chance constraints have been used to address this problem and derive a better evacuation plan where the constraint is satisfied with some specified confidence level. A so-called individual chance constraint can be formulated when the demand parameter S_i in constraint (3b) is replaced by the random demand $\widetilde{S_i}$. Referring to the deterministic model, the demand constraint (3b) is modified to limit the infeasibility of the constraint for each arc by a violation level $\epsilon_i \in (0,1]$.

The chance-constrained model with demand uncertainty can be formulated as follows:

$$\text{Minimize} \quad \sum_{i \in \mathcal{N}_s} \beta_i \qquad \textit{(CCP)} \qquad (4a)$$

$$\text{Subject to:} \quad \mathbb{P}\left(\sum_{p \in \mathcal{P}_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq S_i \right) \geq 1 - \epsilon_i, \qquad \forall i \in \mathcal{N}_s, \qquad (4b)$$

$$\text{(3c)-(3f)}. \qquad (4c)$$

Constraint (4b) is an individual chance constraint equivalent of the deterministic constraint (3b) with the desired probability level imposed individually on each constraint, where parameter $1 - \epsilon_i \in (0,1]$ is the desired reliability level. For modeling with chance constraints, the basic assumption is that the probability distribution function $F_{\tilde{S}_i}$ of the random parameters is known with certainty. When this is the case, and the probabilistic constraint is of the form (4b), a deterministic reformulation is possible:

$$\sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq F_{\tilde{S}_i}^{-1}(1 - \varepsilon_i)\,, \tag{5}$$

where $F_{\tilde{S}_i}^{-1}$ is the inverse distribution function of the random demand.

4 Robust approximation of chance constraints

Assuming the probability distribution of the random demand is known, we can solve the chance-constrained problem as a deterministic model using constraint (5). However, when the distribution is not known, such an approach is not possible. Methods like a min-max approach for a family of distributions can be used instead for situations when the distribution of the demand is not known exactly. We use such an approach via a robust tractable approximation that can be used when only the mean, variance, support, and/or symmetry information is available for the underlying random demand.

We consider the following robust individual chance constraint problem:

$$\text{Minimize} \quad \sum_{i \in \mathcal{N}_s} \beta_i \tag{6a}$$

$$\text{Subject to:} \quad \mathbb{P}\left(\sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq S_i \right) \geq 1 - \epsilon_i, \qquad \forall i \in \mathcal{N}_s, \forall \mathbb{P} \in \wp \tag{6b}$$

$$\text{(3c)-(3f).} \tag{6c}$$

In order to tractably solve model (6) by approximation, we must say more about the family of probability distributions, $\wp$, under consideration. The three propositions that follow specify the family of distributions in three distinct ways and, in turn, provide computationally tractable approximations that ensure the resulting solution is feasible to model (6). Our results are related to those of Calafiore and Ghaoui (2006), but simpler because we only have randomness in the right-hand side. Also, a result similar to the first of our three propositions considering known mean and variance of the underlying distribution can be found in Chung et al. (2012). Our proposition differs from the above published work based on the fact that it is structurally dependent on the SPBM model and derived accordingly. Hereafter, we name the robust model under an uncertain demand distribution as RCCP.

Proposition 1. *For $i \in \mathcal{N}_s$ let the random demand $\tilde{S}_i$ have known mean $\bar{S}_i$ and variance σ_i^2, where we denote this family by $\wp = (\bar{S}_i, \Sigma)$, with $\bar{S} = (\bar{S}_i)_{i \in \mathcal{N}_s}$ and $\Sigma = diag(\sigma_i^2)_{\,i \in \mathcal{N}_s}$, and consider the following model:*

$$\text{Minimize} \quad \sum_{i \in \mathcal{N}_s} \beta_i \qquad (RCCP_1) \tag{7a}$$

Subject to:	$\sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq \sigma_i \sqrt{\frac{1-\epsilon_i}{\epsilon_i}} + \bar{S}_i,$	$\forall i \in \mathcal{N}_s,$	(7b)
	(3c)-(3f).		(7c)

Then every feasible solution of model (7) is feasible for model (6) with $\wp = (\bar{S}_i, \Sigma)$.

Proof. Let (f, β) be a feasible solution of (7). It suffices to show that for each $i \in \mathcal{N}_s$ and $\mathbb{P} \in \wp$:

$$\mathbb{P}\left(\sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i < \tilde{S}_i\right) \leq \epsilon_i \tag{8}$$

From Cantelli's inequality we have a one-sided version of Chebyshev's inequality:

$$\mathbb{P}(\tilde{S}_i \geq \bar{S}_i + \kappa\sigma_\mathrm{i}) \leq \frac{1}{1+\kappa^2}$$

Thus by requiring

$$\sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq \bar{S}_i + \kappa\sigma_\mathrm{i}$$

the probability the chance constraint is violated is no more than $1/1 + \kappa^2$. Setting this value equal to ϵ_i and solving for κ yields the desired result.

Proposition 2. *Assume the hypotheses of Proposition 1 hold. In addition, assume $\bar{S}_i$ is symmetric about its mean, let $\wp = (\bar{S}_i, \Sigma)_S$ denote this family of distributions, and consider the following model:*

Minimize	$\sum_{i \in \mathcal{N}_S} \beta_i$	*(RCCP₂)*	(9a)
Subject to:	$\sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq \sigma_i \sqrt{\frac{1}{2\epsilon_i}} + \bar{S}_i,$	$\forall i \in \mathcal{N}_s,$	(9b)
	(3c)-(3f).		(9c)

Then every feasible solution of model (9) is feasible for model (6) with $\wp = (\bar{S}_i, \Sigma)_S$.

Proof. The proof follows in similar fashion to that of Proposition 1, except it relies on the following one-sided Chebyshev inequality for a random variable that is symmetric about its mean:

$$\mathbb{P}(\tilde{S}_i \geq \bar{S}_i + \kappa\sigma_\mathrm{i}) \leq \frac{1}{2\kappa^2} \tag{10}$$

Setting ϵ_i to the right-hand side of inequality (10) and solving for κ yields the desired result.

Proposition 3. *For $i \in \mathcal{N}_s$ let the random demand $\tilde{S}_i$ have known mean $\bar{S}_i$ and known bounds on its support; i.e., we know $l_i^- < l_i^+$ such that $\mathbb{P}(\tilde{S}_i \in [l_i^-, l_i^+]) = 1$. Let $\wp = (\bar{S}, L)_I$ denote the family of distributions where L contains the intervals $[l_i^-, l_i^+]$, $i \in \mathcal{N}_s$. Consider the following model:*

$$\text{Minimize} \quad \sum_{i \in \mathcal{N}_s} \beta_i \qquad (RCCP_3) \qquad (11a)$$

$$\text{Subject to:} \quad \sum_{p \in \wp_i^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_i \geq (l_i^- - l_i^+) \sqrt{\frac{1}{2} \ln\left(\frac{1}{\epsilon_i}\right)} + \bar{S}_i, \quad \forall i \in \mathcal{N}_s, \qquad (11b)$$

(3c)-(3f). (11c)

Then every feasible solution of model (11) is feasible for model (6) with $\wp = (\bar{S}, L)_I$.

Proof. The proof follows in similar fashion to that of Proposition 1, except it relies on Hoeffding's inequality (Hoeffding 1963) specialized to a single random variable:

$$\mathbb{P}(\tilde{S}_i \geq \bar{S}_i + \kappa\sigma_i) \leq exp\left(\frac{-2\kappa^2}{(l_i^- - l_i^+)^2}\right) \qquad (12)$$

Setting ϵ_i to the right-hand side of inequality (12) and solving for κ yields the desired result.

Depending on whether we are in a situation in which we are willing to assume the demand for each origin node i has: (i) known mean and variance; (ii) known mean and variance and is symmetric; or, (iii) known mean and support, we can apply the respective Propositions 1, 2, or 3 to obtain a tractable robust model, i.e., model (7), (9), or (11). These respective models differ from model (4) with constraint (5)—the case in which the distribution is known—only by the term that appears on the right-hand side of the constraint governing demand satisfaction.

In the proposed robust chance constraint programs (RCCPs), the uncertain demand (S_i) is substituted by an estimation of demand to reach the desired level of confidence (i.e., feasibility of a solution). For instance, having moment information ($\bar{S}_i$, σ_i) and reliability level of ϵ_i, in RCCP1, the source node demand (S_i) is substituted by $\bar{S}_i + \sigma_i\sqrt{1 - \epsilon_i/\epsilon_i}$. Therefore, it becomes important to gather more accurate estimates for the parameters because RCCP will try to generate conservative evacuation plans to protect the evacuees from undesired outcomes such as a longer than expected evacuation time. Although, in most cases, RCCPs may result in an increased CT^*, it is possible to find solutions that are both robust (desired confidence level is guaranteed) and optimal (approximated demands are not inflated enough to increase CT^*) based on propositions 1, 2 and 3. Furthermore, because the proposed RCCPs are linear programs, finding a solution in a timely manner is not an issue.

5 Computational Results

This section is designed to test the performance of the proposed chance constrained approach under various demand scenarios. We further explore the effect of path generation methods on plan performance. Experimental studies are conducted on a test evacuation network shown in Figure 1. We developed our model in a C++ environment and the problem is solved using CPLEX 12.3. Experiments were made on a PC with 3.07 GHz Intel Core i7 processor having 24GB RAM and running Ubuntu 10.04.3.

5.1 Numerical case study

In this network, nodes 1-3 are source nodes and nodes 9-10 are destination nodes. Since the total demand is uncertain, modeling the constraint as a chance constraint and assuming the probability distribution of the perceived demand is known, the solution of the model would result in a feasible evacuation plan for unexpected scenarios within the assumed distribution. However, a problem arises when the assumed distribution is different from the actual distribution. We perform the following numerical tests to show the feasibility of the plan.

For each source node, we assume that the mean of the demand is known with value $\bar{S_i}$ =167, $\forall i \in \mathcal{N}_s$. The standard deviation of the random demand for each of the three source nodes are set to be σ_1 =8.3, σ_2 =7.5 and σ_3 =9.1. Using the chance-constrained model, we assume that demand distribution at each source node is defined by S_i, based on the given statistical parameter values. In the case of uniform distribution, S_i is assumed to be uniform consistent with the given moment information ($\bar{S_i}$, σ_i) of source node i. While for the beta distribution, demand is defined as $S_i = 160+15\tilde{d}_i$ and $\tilde{d}_i$ follows $beta_1(2.68, 3)$, $beta_2(3, 3.45)$ and $beta_3(2.4, 2.75)$, respectively. The capacity of each destination node is set to be 750.

Then, we compare the performance of the chance constraint program (CCP) under uniform (CCP-Unif) and beta distribution (CCP-Beta) assumptions, and robust chance constrained models ($RCCP_1$, $RCCP_2$, and $RCCP_3$). First, an estimated amount of demand for each source node is calculated and is used to solve CCP-Unif, CCP-Beta, $RCCP_1$, $RCCP_2$, and $RCCP_3$. The estimate can be calculated based on (1) the assumed cumulative probability distribution or the given moment information and (2) a given reliability level, i.e., $(1-\epsilon)$. We repeat this computation for the combination of each model and various reliability levels ranging from 60% to 99% probability levels.

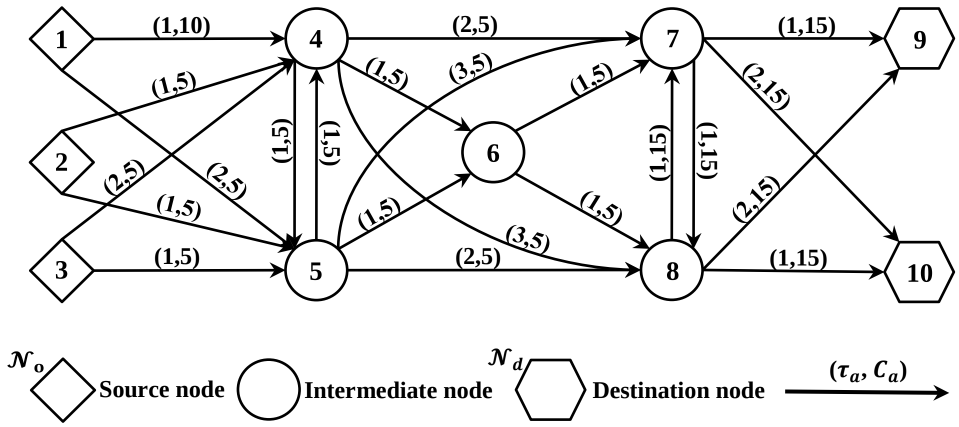

Figure 1: Evacuation test network

After obtaining a plan and optimum clearance time (by using Algorithm 1 in Rungta et al., 2012) for each combination, we apply the Shortfall Determination Algorithm to check the feasibility of the plan. For example, 1,000 demand scenarios are randomly generated using normal, uniform, and beta distributions following given moment information. Assuming for a moment that these scenarios are actual, Step 2 is used to check if the evacuation plan for each source node obtained by CCP-Unif, CCP-Beta, $RCCP_1$, $RCCP_2$, and $RCCP_3$ is feasible. For each source node, we compare a demand scenario (S_{iN}) from an actual distribution (normal, uniform, or beta) with the number of evacuees (S_i) used in the models. Note that the value of S_i changes depending on the optimization model and the corresponding confidence level. If S_{iN} is less than or equal to S_i, then the evacuation plan is considered feasible. Otherwise, the excess demand remains at the source node and the corresponding evacuation plan becomes infeasible.

We compare CCP and RCCP models using two types of performance measures: plan feasibility and optimum clearance time (CT^*).

Table 2: Comparison between CCP and RCCP when the actual demand follows a normal distribution

			Probability Level $(1-\epsilon)$						
			99	98	95	90	80	70	60
CCP-Unif	Demand (S_i)		545	544	541	537	528	520	511
	CT^*		23	23	23	23	23	22	22
	Feas. (Normal)	$\mathcal{N}_1$	95.8%	94.6%	93.1%	91.8%	84.8%	75.9%	63.6%
		$\mathcal{N}_2$	95.0%	95.0%	93.3%	92.1%	86.1%	79.6%	65.0%
		$\mathcal{N}_3$	96.1%	96.1%	94.6%	92.4%	87.9%	79.8%	69.3%
CCP-Beta	Demand (S_i)		520	518	516	513	509	507	504
	CT^*		22	22	22	22	22	22	22
	Feas. (Normal)	$\mathcal{N}_1$	78.5%	78.5%	75.5%	71.3%	67.0%	61.9%	56.7%
		$\mathcal{N}_2$	77.4%	73.4%	73.4%	69.6%	58.4%	58.4%	53.0%
		$\mathcal{N}_3$	77.8%	74.4%	69.6%	65.6%	61.9%	58.2%	53.3%
$RCCP_1$	Demand (S_i)		750	677	611	577	552	540	534
	CT^*		31	28	26	24	24	23	23
	Feas. (Normal)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	98.1%	93.9%	90.7%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	99.9%	96.8%	93.5%	89.7%

$\mathcal{N}_3$	100.0%	100.0%	100.0%	99.9%	98.6%	94.8%	92.0%

Table 2 shows the percentage of demand scenarios that are feasible for each source node when the actual demand follows normal distribution. Each of these values were calculated based on an estimated number of evacuees (S_i) per assumed distribution and confidence level $(1-\epsilon)$ for CCP-Unif, CCP-Beta, $RCCP_1$ models. Furthermore, for given S_i, we can use CCP-Unif, CCP-Beta, $RCCP_1$ models to calculate CT^*, i.e., the time it takes for all evacuees to reach the destination nodes. Similarly, Table 5 and Table 5 in Appendix show the results under the assumption that the actual demand follows Uniform and Beta distribution, respectively.

In Table 2, the feasibility of the tested instances under the $RCCP_1$ approach consistently outperformed the CCP-based approach (i.e., CCP-Beta and CCP-Unif). This is because accuracy of the CCP-based approach depends on the assumed demand distribution, leading to obtain less feasible solutions when the assumed distribution is different from the actual distribution. For example, at the 99% reliability level, we expect that a chance constraint program will provide solutions with 99% feasibility percentage. Instead, CCP-Unif on average produced feasible solutions for only 96.0% of the scenarios (average of 95.8%, 95%, 96.1% which are feasibility percentages of the three source nodes) and CCP-Beta was feasible for only 78.0% of scenarios (average of 78.5%, 77.4%, 77.8%) when the actual distribution (e.g., normal distribution) was different from the assumed beta distribution. In this specific example, the uniform distribution showed better feasibility results than the beta distribution. Similarly, Table 5 and Table 6 in Appendix compare feasibility results of evacuation plans generated by CCP-Unif, CCP-Beta and $RCCP_1$ when actual distribution is uniform and beta, respectively. This brings an important point that the solution of CCP depends on the exact description of the demand distribution.

Shortfall Determination Algorithm

Inputs:

An evacuation network $\mathcal{G}$ consisting of a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$.

An evacuation routing plan (i.e., evacuation paths, flow rates, and schedule for each source node)

Step 1 - Initialization:

for all node $i \in \mathcal{N}_s$ **do**

Generate 1,000 random demand scenarios S_{iN} from the selected underlying distribution (Normal, Uniform or Beta distribution) consistent with the given moment information.

Step 2 - Shortfall determination:

if $S_{iN} \leq S_i$ **then**

Allocate demand proportionately to each selected evacuation path.

Set Feasibility = TRUE.

else if $S_{iN} > S_i$ **then**

Excess demand remains at the source node.

Set Feasibility = FALSE

end if

end for

One can notice from Tables 2, 5, and 6 that RCCP resulted in better feasibility at the cost of higher optimum clearance time. Because the evacuation plan has to be developed without knowing the demand distribution in advance, it is reasonable for the planners to take a conservative approach to give more time for evacuation. However, if the evacuation began much earlier than necessary, the region will suffer from an unnecessary negative economic impact due to loss of work hours. RCCP attempts to address how late we can wait to begin evacuation to meet the desired confidence level under any type of evacuation demand distribution. As the demand of all source nodes increases, the CT^* may go higher. But, RCCP will ensure the resulting CT^* is optimal for available data for demand estimate (see propositions 1, 2 and 3).

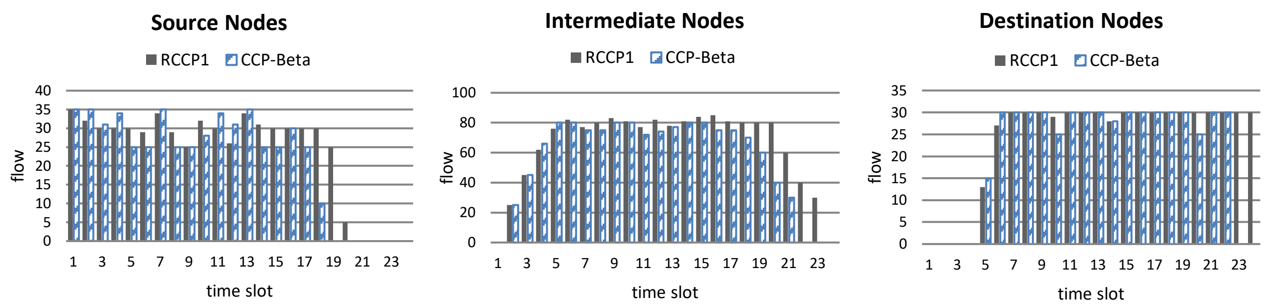

Figure 2: Flow passing through nodes over the course of an evacuation (for $(1-\epsilon) = 90\%$*)*

We further compare results of a robust model ($RCCP_1$) with a chance-constraint model (CCP-Beta) to see how their scheduled flows behave over the course of an evacuation. Figure 2 displays snapshots of flows (evacuees) at different times (discrete time intervals) that pass through different source nodes, intermediate nodes, and destination nodes at the 90% confidence level in the models. More detailed results for various confidence levels are tabulated in Table 7 and Table 8 in Appendix. Looking at the flow on destination nodes, we observe that the CCP-Beta had its last flow at time 22 compared to time 24 for $RCCP_1$. Based on the previous results, plans using a RCCP expect a longer time to complete the evacuation as compared to the plans provided by CCPs. However, looking back at the scenarios projected in Table 2, 5 and 6, there is a good chance that the CCP-based plans may not feasible when the actual demand is different from what it was assumed in the planning stage. For the case of case of CCP-Beta with confidence level of 90%, when the actual (or realized) demand follows a normal distribution, the provided plan was feasible only 68.83% of the scenarios tested (the average of 71.3%, 69.6% and 65.6% corresponding to source nodes $\mathcal{N}_1, \mathcal{N}_2,$ and $\mathcal{N}_3$). This can result in a devastating consequence; the feasibility could drop from the aimed 90% to a mere 68.83%. In reality, having an accurate estimation on demand distribution is near impossible. Hence, CCPs may not properly provide reliable plans for emergency evacuations. Although RCCPs project a longer CT^*, the resulting plans are expected to be more reliable.

It is possible to collect data that can give partial information about the distribution beyond first and second moments. In the following we examine the results when extra information regarding the demand distribution such as symmetry and support information is available. Shortfall Algorithm is used again and we follow the same experimental procedure as described previously. First, the results are obtained based solely on the information of the first two moments of the random demand data (Model $RCCP_1$). The next experiments are based on the assumption that the distribution is symmetric (Model $RCCP_2$). The third set of experiments is based on the assumption that the mean of the demand is known and the deviation of the demand from mean for each source is bounded (Model $RCCP_3$). Tables 3, 9 and 10 show the results when the actual demand is drawn from, respectively, normal, uniform and beta distributions, and robust approximation of the chance constraint is performed using the known statistical information of the demand distribution (Table 9 and 10 in Appendix).

As expected, results shown in Tables 3, 9 and 10 suggest that a tighter approximation can be made when we have more information about the distribution. For example, for a desired reliability of 99% for the constraint to be feasible, the demand approximation based on the support information ($RCCP_3$) gives the tightest bound and results in a shorter CT^*. Information about the symmetry of the distribution significantly improves the tightness of the approximation and makes the demand approximation tighter resulting in an efficient evacuation plan. Note that computation times for solving RCCP and CCP models for the test network were less than a second.

Table 3: Performance of RCCP under various distribution information (true demand distribution: Normal)

			Probability Level $(1-\epsilon)$						
			99	98	95	90	80	70	60
Moment ($RCCP_1$)	Demand (S_i)		750	677	611	577	552	540	534
	CT^*		31	28	26	24	24	23	23
	Feas. (Normal)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	98.1%	93.9%	90.7%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	99.9%	96.8%	93.5%	89.7%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	99.9%	98.6%	94.8%	92.0%
Symmetry ($RCCP_2$)	Demand (S_i)		679	627	581	558	542	534	531
	CT^*		28	26	25	24	23	23	23
	Feas. (Normal)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	98.9%	94.9%	89.9%	87.4%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	98.9%	95.5%	91.4%	89.1%
		$\mathcal{N}_3$	100.0%	100.0%	99.8%	98.6%	94.6%	90.1%	87.7%
Support ($RCCP_3$)	Demand (S_i)		595	588	577	568	557	549	543
	CT^*		25	25	25	24	24	24	23
	Feas. (Normal)	$\mathcal{N}_1$	100.0%	99.9%	99.9%	99.5%	98.8%	98.0%	95.3%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	99.5%	98.5%	97.4%	94.5%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	99.9%	99.6%	98.9%	97.1%

We can conclude that, although RCCP recommends a longer optimum clearance time in evacuation planning, it provides a conservative solution that will work under unknown demand distribution. While planning for an evacuation, if complete information of the demand distribution is available, then the tightest approximation can be obtained. Nevertheless, it is arguably more sensible to use the robust approximation to come up with an evacuation plan when complete information of the distribution is not known. As in such cases, an incorrect assumption would lead to undesired infeasible plans. Insights learned from these results can greatly help evacuation managers to prepare well for an upcoming disaster.

5.2 Path enumeration impact on evacuation plan performance

As we mentioned earlier, it is computationally intractable to find an optimal set of evacuation paths, flow assignment on the paths, and schedule over an optimal clearance time for a large-scale evacuation problem. The premise of our proposed approach was to ease the computational burden of dealing with large-scale evacuation networks. This is possible by using the path-based approach (Rungta et al., 2012). The solution quality of the path-based approach depends on the number of pre-defined candidate paths as an input to the evacuation plan optimization model. Having a sufficient number of the candidate paths is important to obtain a reliable plan. We should note that as we increase the number of candidate paths to be fed to the optimization model, its computational time can increase due to the increased number of variables.

In this section, numerical studies are conducted to analyze the performance of the path-based model for different initialized number of paths for the model. Computational times for solving the small test network in Figure 1 are less than one second. To establish a benchmark for experiments of this section, all 216 paths of the previous network have been enumerated and used in $RCCP_1$ to derive optimum evacuation plan and clearance time. Estimation of the demand that is used for this experiment is derived from the $RCCP_1$ with 99% reliability level meaning that the right hand side of constraint (7b) takes values 182, 180 and 183 for source nodes 1, 2 and 3, respectively.

Then result of the model is obtained when less number of paths are used as an input of $RCCP_1$ and by comparing with the benchmark plan, their feasibility is calculated. As it is shown in Table 4, even when only 10 shortest paths per node are used in the model (total 30 input paths), feasibility for all source nodes are 100% and performance of it is the same as the optimum plan. In cases that number of input paths are less than 10 paths per node, feasibility might deviate from 100%. For instance, when 10, 9 and 9 paths are selected for nodes 1, 2 and 3 respectively, the feasibility for the first and third node drops to 93.4%, and 65.6% and overall feasibility of the plan decreases to 86.2%. From the results, we can conclude that we can reach global optimal solutions for the network by only enumerating 30 paths instead of using all 216 paths. Therefore, using the path-based method in the proposed RCCP models makes them capable to deal with large-scale networks through using less number of paths while obtaining optimal solutions.

Table 4: Evacuation percentage vs. the number of enumerated paths

nPaths (pcnt) per Source Node			CT^*	Total *nPaths* (pcnt)
$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$		
30 (100%)	30 (100%)	30 (100%)	23	90 (100%)
10 (100%)	10 (100%)	10 (100%)	23	30 (100%)
10 (100%)	9 (100%)	10 (100%)	23	29 (100%)
10 (93.4%)	9 (100%)	9 (65.6%)	26	28 (86.2%)
10 (93.4%)	10 (91.7%)	9 (73.8%)	26	29 (86.2%)
9 (96.2%)	9 (94.4%)	10 (65.8%)	27	28 (85.3%)

9 (89.6%)	10 (98.9%)	10 (66.7%)	27	29 (85.0%)
9 (85.2%)	10 (91.7%)	9 (32.8%)	32	28 (69.7%)
9 (96.2%)	9 (58.3%)	9 (54.6%)	32	27 (69.7%)
7 (98.9%)	7 (41.7%)	7 (68.3%)	32	21 (69.7%)
5 (60.4%)	5 (86.1%)	5 (62.8%)	32	15 (69.7%)
3 (98.9%)	3 (5.6%)	3 (51.9%)	41	9 (52.3%)

nPaths: number of paths; pcnt: Percent feasibility

In order to better study the influence of path enumerations, computational experience is implemented on a large metropolitan area evacuation network. Figure 3 represents the transportation network of Houston, Texas, the fourth largest city in the US. Houston has experienced many hurricanes and is one of the most vulnerable metropolitan cities situated on the Gulf coast. In the considered evacuation network, there are a total of 42, including 13 source nodes and 4 destination nodes. Data used for this network is obtained from the work of (Rungta et al., 2012 and Lim et al., 2016).

The total mean demand on source nodes is 566,000 and transit times are multiplies of $\tau = 30$ minute intervals. Demand is not uniformly distributed among source nodes and the average number of demand on source nodes 1-6, 7-10 and 11-13 are 100, 3500 and 14000, respectively. Due to this variety in the amount of demand of source nodes, in order to better compare the performance of path enumerations, we define three levels of source nodes. Level 1 includes nodes $\mathcal{N}_1$ -$\mathcal{N}_6$ which have the least amount of demand, level 2 includes source nodes $\mathcal{N}_7$ -$\mathcal{N}_{10}$ and level 3 contains nodes $\mathcal{N}_{11}$ -$\mathcal{N}_{13}$ with highest amount of demand.

For the third level of nodes, since the amount of demand compared to other classes is considerable, we consider three number of path levels: Small (5-10 paths), Medium (10-25) and Large (60-90 paths). For the first and second level of nodes ($\mathcal{N}_1$ -$\mathcal{N}_6$ and $\mathcal{N}_7$ -$\mathcal{N}_{10}$), we consider Medium and Large levels. Therefore, our experiment is conducted by evaluating results of each $2 \times 2 \times 3 = 12$ combination of source node level and path number level.

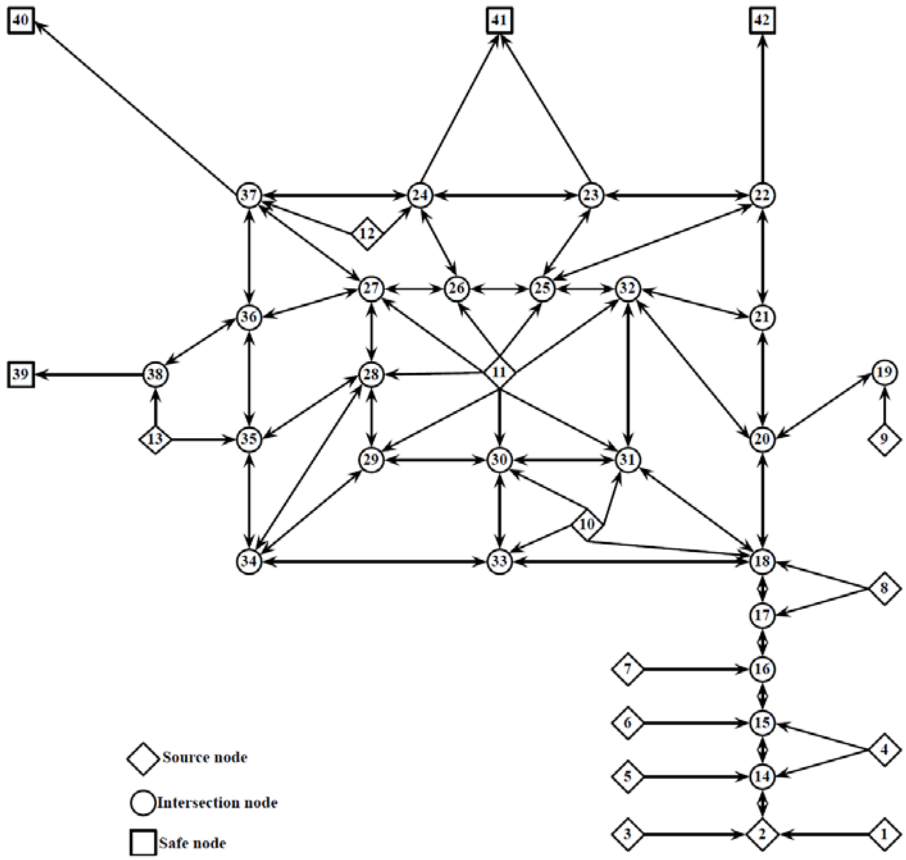

Figure 3: City of Houston Transportation Network

In Table 11 (see Appendix), mean and standard deviation for the number of input paths as well as feasibility percentage and solution time for the combination of each source node level and path number level are presented. In combination $C_1(M,M,L)$, highest levels for number of paths (Medium, Medium and Large levels) are considered for node levels $\mathcal{N}_1$ -$\mathcal{N}_6$, $\mathcal{N}_7$ -$\mathcal{N}_{10}$, and $\mathcal{N}_{11}$ -$\mathcal{N}_{13}$. Hence, results of $C_1(M,M,L)$ is selected as a benchmark to evaluate the results of other combinations. As shown, when Large and Medium path levels are selected for source nodes 11 to 13 ($\mathcal{N}_{11}$ -$\mathcal{N}_{13}$), regardless of the path levels of other two node levels, 100% feasibility is achieved. Only when the number of paths for populated source nodes $\mathcal{N}_{11}$ -$\mathcal{N}_{13}$ are within the Small level (5-10 paths), as is in C_6, C_9, and C_{12}, feasibility of the plan deviates from 100%. But in other cases, 100% feasibility is achieved by using a fewer number of paths. Having an enough number (i.e., more than 10) of pre-defined paths for source nodes 11 to 13 seem to ensure 100% feasibility. However, as the number of pre-defined paths in the model increases, the solution time increases at a slow pace. Hence, it can adequately handle large-scale evacuation networks. Other benefits of the path-based RCCP models (as compared to arc-based models) are that it simplifies the optimization process by performing path formation prior to flow assignment and schedule, and it is easy to implement the plan in practice.

6 Conclusion

While most of the existing evacuation traffic assignment problems assume deterministic input parameters (such as demand), there are a few works that account for uncertainties in parameters. Stochastic programming techniques, specifically chance-constrained programming, are usually employed to come up with a reliable evacuation plan. However, in the context of mass evacuation, only partial information of demand distribution may be known as opposed to the exact distribution. To address this challenge, we have developed a stochastic approach where only partial information of demand distribution (i.e., moment, support, or symmetry) is available. To overcome the computational burden of a stochastic model, robust approximations of chance-constrained problems are provided to model traffic demand uncertainty in evacuation networks. Particularly, when the demand variable has an arbitrary distribution in the evacuation problem, we propose the utilization of a distribution-free linear approximation technique to solve the problem. Furthermore, three models (RCCP) are proposed by deriving robust linear approximations of the demand-satisfaction constraint for the cases that moments, support and symmetry properties of the distribution are known. Performance of the proposed RCCP models were compared with a chance constraint programming model under the assumption that the demand distribution follows uniform or beta distributions (CCP-Unif, CCP-Beta). Feasibility of each of these five models were measured when the real demand distribution was assumed to follow normal, uniform or beta. Results showed that the solution of the CCP-based approach depends on the exact description of the demand distribution, and the probability of not being able to complete evacuation is high when the real demand distribution is different from what was assumed. This issue was clearly overcome by using RCCP Models. RCCPs provide better feasible plans regardless of the underlying assumed distribution. We also note that among RCCP models, $RCCP_3$ provide more efficient evacuation plans in the context of both feasibility and optimum amount for clearance time as more information about the demand (support information) is being considered compared to the other two cases.

One can extend this work by incorporating other types of parameter uncertainty such as road capacity or travel times during the planning phase, which will give a more helpful solution to evacuation managers. However, when there are more probabilistic parameters in the model, it becomes computationally more challenging to solve the resulting model. Therefore, computationally more efficient solution approaches will be always desired for the models to be practically useful. Furthermore, an investigation on approaches to better control the level of conservatism of the PBM solution (see Bertsimas and Sim, 2004) can be another venue for potential future research.

Appendix

Table 5: Performance comparison between CCP and RCCP (true distribution: uniform)

	Probability Level $(1-\epsilon)$

			99	98	95	90	80	70	60
CCP-Unif	Demand (S_i)		545	544	541	537	528	520	511
	CT^*		23	23	23	23	23	22	22
	Feas. (Unif)	$\mathcal{N}_1$	100.0%	98.3%	95.4%	92.7%	82.0%	72.1%	62.2%
		$\mathcal{N}_2$	100.0%	100.0%	95.5%	91.4%	79.7%	71.7%	58.6%
		$\mathcal{N}_3$	100.0%	100.0%	97.7%	91.9%	82.7%	73.3%	65.0%
CCP-Beta	Demand (S_i)		520	518	516	513	509	507	504
	CT^*		22	22	22	22	22	22	22
	Feas. (Unif)	$\mathcal{N}_1$	71.6%	71.6%	68.1%	63.9%	61.0%	58.2%	54.9%
		$\mathcal{N}_2$	72.9%	69.8%	69.8%	66.9%	57.7%	57.7%	53.0%
		$\mathcal{N}_3$	73.0%	70.8%	67.9%	64.6%	60.5%	56.9%	51.9%
RCCP₁	Demand (S_i)		750	677	611	577	552	540	534
	CT^*		31	28	26	24	24	23	23
	Feas. (Unif)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	94.5%	88.1%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	97.2%	88.4%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	95.3%	88.9%

Table 6: Performance comparison between CCP and RCCP (true distribution: beta)

			Probability Level $(1-\epsilon)$						
			99	98	95	90	80	70	60
CCP-Unif	Demand (S_i)		545	544	541	537	528	520	511
	CT^*		23	23	23	23	23	22	22
	Feas. (Beta)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	99.2%	83.4%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	99.3%	84.2%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	99.8%	89.6%
CCP-Beta	Demand (S_i)		520	518	516	513	509	507	504
	CT^*		22	22	22	22	22	22	22
	Feas. (Beta)	$\mathcal{N}_1$	98.5%	98.5%	95.6%	90.8%	83.2%	73.7%	63.1%
		$\mathcal{N}_2$	99.0%	96.8%	96.8%	92.0%	76.2%	76.2%	66.7%
		$\mathcal{N}_3$	100.0%	98.7%	95.2%	91.7%	84.1%	75.2%	64.4%
RCCP₁	Demand (S_i)		750	677	611	577	552	540	534
	CT^*		31	28	26	24	24	23	23
	Feas. (Beta)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Table 7: Amount of RCCP₁'s flow that reaches to each node over the course of evacuation, $(1-\epsilon) = 90\%$

	Time slot																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$\mathcal{N}_1$	15	12	10	10	10	9	14	9	5	12	10	6	14	11	10	10	10	10	5					
$\mathcal{N}_2$	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10					
$\mathcal{N}_3$	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	5				
$\mathcal{N}_4$		15	20	20	19	20	17	19	20	20	18	20	19	20	20	20	20	20	20	20	10			
$\mathcal{N}_5$		10	15	17	15	17	15	17	19	16	15	18	15	15	19	20	20	20	20	20	10			
$\mathcal{N}_6$			10	10	10	10	10	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10		
$\mathcal{N}_7$				10	15	17	18	17	18	17	18	16	18	16	19	16	15	15	15	15	15	15	15	
$\mathcal{N}_8$				5	17	18	17	18	16	18	16	18	16	20	16	19	16	15	15	15	15	15	15	
$\mathcal{N}_9$					8	12	15	15	15	15	15	15	15	13	15	15	15	15	15	15	15	15	15	15
$\mathcal{N}_{10}$					5	15	15	15	15	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15

Table 8: Amount of CCP-Beta's flow that reaches to each node over the course of evacuation, $(1-\epsilon) = 90\%$

	Time slot																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
$\mathcal{N}_1$	15	15	15	14	5	5	15	5	5	8	14	15	15	5	5	10	5					
$\mathcal{N}_2$	10	10	6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	5				
$\mathcal{N}_3$	10	10	10	10	10	10	10	10	10	10	10	6	10	10	10	10	10	5				
$\mathcal{N}_4$		15	20	20	20	20	20	20	20	20	17	19	20	20	20	20	20	15	10			
$\mathcal{N}_5$		10	15	16	20	20	15	15	20	15	15	15	19	20	20	15	15	15	10			
$\mathcal{N}_6$			10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10		
$\mathcal{N}_7$				10	15	15	15	15	15	15	15	15	13	15	15	15	15	15	15	15	15	
$\mathcal{N}_8$				10	15	15	15	15	15	20	15	15	15	15	15	15	15	15	15	15	15	
$\mathcal{N}_9$					10	15	15	15	15	10	15	15	15	13	15	15	15	15	15	10	15	15
$\mathcal{N}_{10}$					5	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15

Table 9: Performance of RCCP (true distribution: Uniform)

			Probability Level $(1-\epsilon)$						
			99	98	95	90	80	70	60
Moment (RCCP1)	Demand (S_i)		750	677	611	577	552	540	534
	CT^*		31	28	26	24	24	23	23
	Feas. (Unif)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	94.5%	88.1%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	97.2%	88.4%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	95.3%	88.9%
Symmetry (RCCP2)	Demand (S_i)		679	627	581	558	542	534	531
	CT^*		28	26	25	24	23	23	23
	Feas. (Unif)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	98.5%	88.7%	84.0%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	97.1%	89.5%	85.1%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	97.8%	87.8%	84.7%
Support (RCCP3)	Demand (S_i)		595	588	577	568	557	549	543
	CT^*		25	25	25	24	24	24	23
	Feas. (Unif)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	99.0%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	96.7%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Table 10: Performance of RCCP (true distribution: Beta)

			Probability Level $(1-\epsilon)$						
			99	98	95	90	80	70	60
Moment (RCCP1)	Demand (S_i)		750	677	611	577	552	540	534
	CT^*		31	28	26	24	24	23	23
	Feas. (Beta)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Symmetry (RCCP2)	Demand (S_i)		679	627	581	558	542	534	531
	CT^*		28	26	25	24	23	23	23
	Feas. (Beta)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
		$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
		$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Support (RCCP3)	Demand (S_i)		595	588	577	568	557	549	543
	CT^*		25	25	25	24	24	24	23
	Feas. (Beta)	$\mathcal{N}_1$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

$\mathcal{N}_2$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
$\mathcal{N}_3$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Table 11: Evacuation percentage vs. the number of enumerated paths on the Houston evacuation network

	CT^*	Computation time (s)	Number of paths	(Mean, STD)	Source Nodes $\mathcal{N}_1$ - $\mathcal{N}_6$	$\mathcal{N}_7$ - $\mathcal{N}_{10}$	$\mathcal{N}_{11}$ - $\mathcal{N}_{13}$
$C_1(M,M,L)$	147	1.05	395	Paths	(16, 5)	(21, 4)	(72, 9)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_2(M,M,M)$	147	0.33	221	Paths	(15, 5)	(16, 5)	(23, 1)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_3(M,M,S)$	147	0.28	176	Paths	(14, 3)	(17, 5)	(9, 1)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_4(M,S,L)$	147	0.67	332	Paths	(17, 5)	(9, 2)	(65, 6)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_5(M,S,M)$	147	0.27	154	Paths	(14, 5)	(8, 2)	(13, 3)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_6(M,S,S)$	202	0.31	168	Paths	(19, 7)	(7, 1)	(8, 3)
				Feasibility	(100%, 0%)	(74.6%, 38.5%)	(69.2%, 32.0%)
$C_7(S,M,L)$	147	0.45	337	Paths	(8, 2)	(19, 5)	(72, 9)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_8(S,M,M)$	147	0.24	170	Paths	(9, 1)	(17, 5)	(16, 4)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_9(S,M,S)$	191	0.19	145	Paths	(8, 2)	(20, 3)	(7, 2)
				Feasibility	(66.7%, 51.6%)	(96.8%, 6.4%)	(70.1%, 31.1%)
$C_{10}(S,S,L)$	147	0.32	269	Paths	(7, 2)	(8, 2)	(66, 8)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_{11}(S,S,M)$	147	0.19	126	Paths	(8, 2)	(8, 2)	(16, 8)
				Feasibility	(100%, 0%)	(100%, 0%)	(100%, 0%)
$C_{12}(S,S,S)$	241	0.16	96	Paths	(7, 2)	(7, 1)	(5, 1)
				Feasibility	(100%, 0%)	(80.7%, 38.6%)	(61.7%,19.6%)

*L, S and M stands for Large, Medium and Small level for number of paths

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