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Dynamic Network Flow Optimization for Real-time Evacuation Reroute Planning under Multiple Road Disruptions

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Abstract

During the course of an evacuation, evacuees often encounter unexpected incidents interrupting their plans for evacuation. Roads may not be accessible due to flooding, wild-fire propagation, accidents, the collapse of highway structures, and various other reasons. The evolving disturbances to the evacuation plan due to road disruptions may prolong the evacuation process and lead to chaos, injuries, and loss of life unless a quick, efficient recovery plan is implemented. In this work, we aim to provide a rerouting approach for an evacuation network that undergoes road disruptions. Unlike previous studies, it is assumed that incidents can occur on multiple roads and that the time of each occurrence can differ from the time of other occurrences. Flow optimization techniques are used to represent evacuation traffic flow on the transportation network. A dynamic traffic flow rate is considered in which the evacuation flow rate can change over time during the planning horizon. The variation in the flow rates enables a better projection of the traffic dynamics and consequences caused by disturbances. Furthermore, a path-based dynamic network flow optimization formulation is proposed to make the model scalable for large evacuation networks. Two preprocessing algorithms are introduced to calculate specific parameters associated with road disruptions and topology of the evacuation network. The use of these parameters enables us to transform the original optimization model into a linear model to reduce the computational burden. Numerical experiments are made to show the performance of the proposed model. Furthermore, the effects of specific features such as disruption time, disturbance location, and the plan updating time on the evacuation process are investigated. Results indicate that when more incidents occur later or when incident information is received earlier, the magnitude of the rerouting completion time is lessened.

Keywords: Short-notice evacuation, dynamic network flow problem, network disruption.

1 Introduction

Every year, numerous hazardous incidents have affected millions of people worldwide (see Figure 1). Natural hazards are the common concern for many communities, including Geo-physical (Geological) incidents, such as earthquakes, volcanic disruptions, and tsunamis, or weather-related events, such as storms, hurricanes, droughts, and tornados. Various case-scenarios regarding technological failures and intentional malevolence, such as nuclear meltdowns, hazardous material spills, and terrorist attacks, can also wreak havoc on communities.

One of the most critical elements in response to a major disaster is the evacuation of people in the affected area, as it is directly associated with protecting human lives. An evacuation refers to the mass physical movement of people from endangered areas to safe shelters prior to the onset of, during, or after a hazardous event. According to a 2005 report by the Nuclear Regulatory Commission (2005), the need for evacuation of 1,000 or more Americans rises about every three weeks. In another report, FEMA (2008)

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declared that approximately 45 to 75 major evacuations occur annually in the United States. The federal government directs state officials to provide evacuation plans using guidelines such as the Robert T. Stafford Disaster Relief and Emergency Assistance Act (Robert, 2000) and the FEMA Comprehensive Planning Guide 101 (FEMA, 2010). Meanwhile, as a result of variations in state populations, geographic characteristics, and transportation infrastructure, evacuation plans provided in different states tend to differ from one another.

There are two phases of evacuation planning and management. The first phase is to develop a route plan and schedule for the evacuation prior to the arrival of an adversarial event. Decisions regarding route assignments, time and schedule of evacuees' departure, resource allocation, and the operations planning horizon should be made during this phase. Such a process entails assessing the topology of the transportation network, available human resources and facilities, dynamics of traffic flow, and road congestion level. The second phase, which is the focus of this paper, monitors the progress of the evacuation, detects deviations from the plan, and makes adjustments, if needed, based on real-time information. Therefore, it is critical to have precise information about the real-time progress of evacuation and transportation infrastructure, including the residual capacity of the roads in the evacuation network.

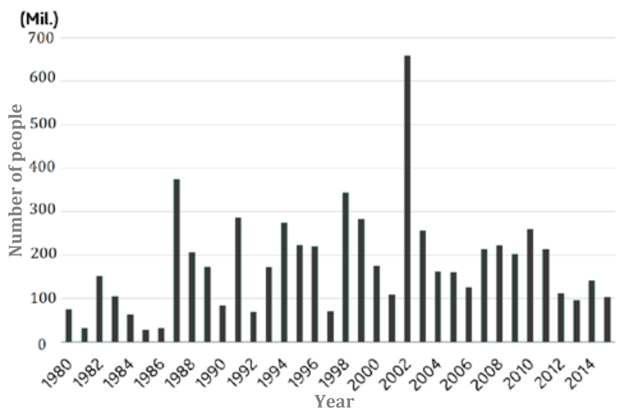

Figure 1: People affected by hazards each year (Source: EM-DAT)

During the course of an evacuation, the capacity of the roads can be blocked or severely compromised due to the occurrence of many unforeseen events. Faturechi and Miller-Hooks (2015) presented a comprehensive literature review related to the performance of transportation systems in the face of hazardous events, the potential physical damages to the infrastructure, its vulnerability, reliability, robustness, flexibility, survivability, and resilience. Roads may become impassable due to flooding, sinkholes, railway barriers at railroad crossings, debris fallen on road surfaces, the collapse of highway structures caused by high winds, and subways submerged with storm water. These Real-Time Events (RTE) (Beroggi and Wallace, 1995) can disrupt the evacuation plan at hand and create an unreasonable amount of traffic congestion. Furthermore, RTEs can heavily jeopardize the safety of evacuees and yield high ground delays. Timely and effective decisions on modifying the initial route plan to have a full, fast, and complete evacuation can help alleviate the risk of chaos and prolonged delays. Therefore, the purpose of our study is to provide emergency managers with a real-time recovery scheme (i.e., new evacuation routes and schedule) in the case of a transportation network road disruption. We propose to develop such plans in a way that the total time required to clear the network is minimized.

To date, various strategies and methods have been proposed to develop efficient evacuation plans. Both simulation methods (Sheffi et al. 1981, Pidd et al. 1996, De Silva and Eglese 2000) and optimization techniques (Kim et al. 2008, Rungta et al. 2012, Lim et al. 2016, and Lim et al. 2019) have been widely used to study and improve evacuation decisions along with predictions pertaining to the minimum amount of evacuation completion time. Investigating a transportation network performance in the face of disastrous events has been studied by several authors. Lin et al. (2012) analyzed the performance of a stochastic-flow network under multiple correlated failures of physical lines and routers. Chen and Miller-Hooks (2012) introduced a network resilience metric to measure recovery capability of a freight transportation network under the possibility of disruptions. Bier and Hausken (2013) investigated the impact of blockage of specific roads of transportation networks due to intentional attacks. Faturechi and Miller-Hooks (2014) provided a bi-level three-stage stochastic program in which response strategies are defined in the upper level to enhance the network resilience and the affected users choose best possible routes for themselves in the lower level based on the upper-level decisions.

Our literature review reveals that the problem of the real-time plan adjustment for evacuation has received significantly less scholarly attention. Lim et al. (2016) provided a preprocessing algorithm that utilizes a path-based network flow optimization approach to reassign paths for evacuees affected by an incident, assuming the availability of real-time traffic information. However, there are two limitations on their work. First, their model is limited to a single road incident. Second, evacuation vehicles are assumed to join the evacuation routes at a constant flow rate. Both assumptions are less realistic because multiple road disruptions can occur at any time, and the vehicle flow rate joining major evacuation routes can vary over the evacuation planning horizon. Therefore, the purpose of this paper is to provide an approach to relax both of the assumptions within the context of a dynamic network flow optimization, in which a variable vehicle flow rate is assigned to each time interval of the planning horizon, and disruptions can occur on multiple roads of the network. The number of evacuees leaving the intermediate nodes into new pathways in the network can vary in each time interval, which results in a more realistic representation of evacuation flow dynamics and congestion.

The paper is organized as follows. Section 2 provides a detailed description of the problem assumptions and the developed dynamic network flow optimization model formulation. Section 3 introduces preprocessing algorithms to calculate specific parameter values and a bi-section algorithm to calculate the evacuation completion time. In Section 4, different disruption scenarios are discussed, and the performance of the proposed approach is investigated both on a sample network and on a real, large-scale metropolitan evacuation network. The same section discusses computational results to examine the effects of the incident time and incident location along with the plan updating time. Section 5 concludes the paper with a short summary and a future research direction.

2 Problem Statement

Traffic flow and congestion on the evacuation network are represented through a dynamic network flow problem. A dynamic network is composed of multiple static networks in which each static network depicts the status of the network at a specific time (Ford Jr and Fulkerson, 2015). Let us consider a directed network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ consisting of a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$ (see Figure 2). The nodes are categorized as origin nodes ($\mathcal{N}_o$), intermediate nodes, and destination nodes ($\mathcal{N}_d$). Intermediate nodes typically represent intersections of roads in the evacuation network. The planning horizon is divided into discrete time intervals represented by set $\mathbb{T} = \{0, 1, \ldots, \mathrm{T}\}$. A traffic assignment on this network relies upon a representation of traffic as a series of vehicle flows at each time interval as per topology of the network. The transit time of arc $a \in \mathcal{A}$ is shown by τ_a, and the time it takes to reach arc $a \in \mathcal{A}$ from the origin of path $p \in \wp$ is denoted by θ_{pa}.

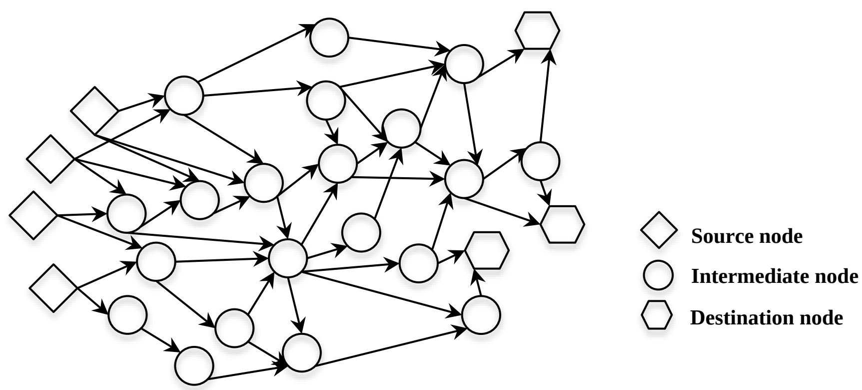

Figure 2: A directed graph representing an evacuation network

Our approach is based on a path-based approach (see Appendix A) to expedite solving the evacuation rerouting problem. In a path-based approach, all possible paths between each origin-destination (O-D) of the evacuation network are generated and enumerated *a priori,* and candidate paths are selected and defined as an input (i.e., set $\wp$) to the optimization model. Consequently, it reduces the number of variables and constraints in the optimization model and reduces the computational complexity of optimizing evacuation planning models. of the model. This is an advantage over other well-known methods such as a Cell Transmission Model (CTM) in simulating traffic speed and congestion (Lim et al. 2019).

Parameter DT_a is used to denote the disruption time on each arc $a \in \mathcal{A}$. The aim of the rerouting path-based model ($RPBM$) is to optimally utilize the residual capacity of the road networks to reassign the disturbed evacuees to new paths such that the overall evacuation reroute completion time is minimized.

The $RPBM$ assigns the disturbed flow to both unaffected paths as well as partially damaged paths, or called *affected paths*. An affected path is a path involving one or more disrupted arcs. Although a path can be interrupted, the residual capacity of its intact arcs can still be used to reroute the flow and push it forward through the evacuation network. When the reassigned flow reaches a damaged arc of an affected path, it will gather and wait behind the associated node and can be sequentially rerouted to a path at another time (see Figure 3).

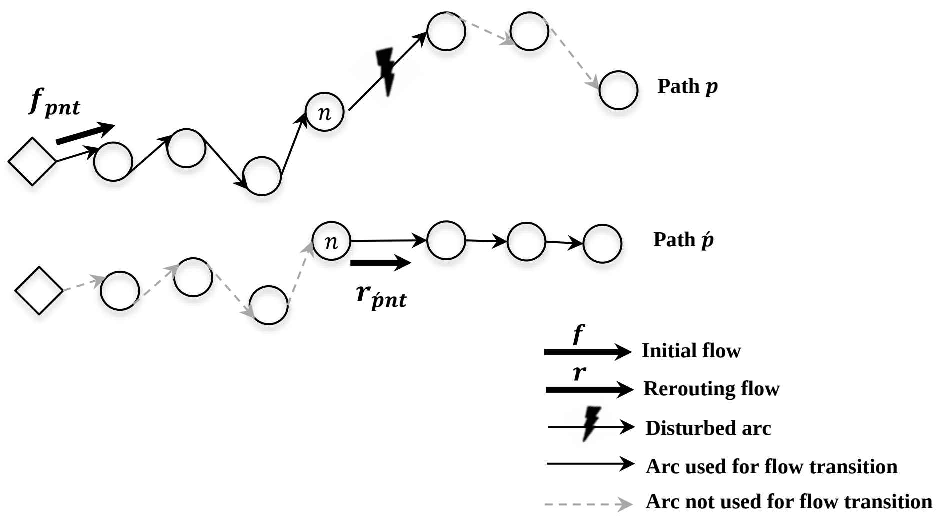

Figure 3. Rerouting in RPBM

For developing the mathematical formulation, the following notation is used:

Sets:

$\mathcal{N}$	Set of all nodes
$\mathbb{T}$	Set of all time slots
$\wp$	Set of all paths

Decision Variables:

r_{pnt}	Disturbed flow of node $n \in \mathcal{N}$ that is rerouted and assigned to alternate route p at time $t \in \mathbb{T}$
hc_{nt}	Remaining disrupted flow on node $n \in \mathcal{N}$ at time $t \in \mathbb{T}$

Parameters:

f_{pt}	Flow that starts on path $p \in \wp$ at time $t \in \mathbb{T}$ (pre-disruption plan)
H_{pnt}	Disturbed flow on node $n \in \mathcal{N}$ of path $p \in \wp$ at time $t \in \mathbb{T}$
θ_{pa}	Transit time from the origin of path $p \in \wp$ to arc $a \in \mathcal{A}$
C_a	Capacity of arc $a \in \mathcal{A}$
D_n	Demand of source node $n \in \mathcal{N}$
ℓ_n	Capacity of destination node $n \in \mathcal{N}$
τ_a	Transit time on arc $a \in \mathcal{A}$
L_{pn}	Takes value 1 if node $n \in \mathcal{N}$ is the source node of path $p \in \wp$, and otherwise 0
K_{pn}	Takes value 1 if node $n \in \mathcal{N}$ is the destination node of path $p \in \wp$, and otherwise 0
$\acute{\theta}_{pn}$	Transit time from the origin of path $p \in \wp$ to node $n \in \mathcal{N}$
δ_{pa}	Takes value 1 if arc $a \in \mathcal{A}$ belongs to path $p \in \wp$, and otherwise 0
γ_{na}	Takes value 1 if node $n \in \mathcal{N}$ is the upstream (origin) node of arc $a \in \mathcal{A}$, and otherwise 0
φ_{pmn}	Takes value 1 if node $n \in \mathcal{N}$ is not behind node $m \in \mathcal{N}$ on path $p \in \wp$, and otherwise 0
W_{pnt}	Takes value 1 if the flow on path $p \in \wp$ starting at time $t \in \mathbb{T}$ reaches the merging arc from node $n \in \mathcal{N}$ before the disruption time of the arc

V_{pnt}	Takes value 1 if the flow on path $p \in \wp$ starting at time $t \in \mathbb{T}$ is disturbed and stuck behind node $n \in \mathcal{N}$, and otherwise 0
η_{ptmn}	Takes value 1 if the flow on path $p \in \wp$ starting at time $t \in \mathbb{T}$ is not affected through node $m \in \mathcal{N}$ and also is not disturbed between node $m \in \mathcal{N}$ and node $n \in \mathcal{N}$ (m is behind n), and otherwise 0
∂_{pn}	Takes value 1 if there is no disturbed arcs on path $p \in \wp$ after node $n \in \mathcal{N}$, and otherwise 0

Mathematical properties of the *PBM* model do not allow direct representation of flow departing from intermediate nodes. Variables denoting the flow are always related to the evacuees leaving the *source node* of a path rather than the *intermediate node*. However, a method is needed for rerouting the flow in order to reflect the flow departing from an intermediate node (see Figure 4). To resolve the issue in the optimization model formulation, the rerouting variable r_{pnt} is introduced to denote the amount of flow departing from the origin of path $p \in \wp$ at time $t \in \mathbb{T}$. Nevertheless, in our constraints, we ignore the values of r_{pnt} associated with preceding nodes (or arcs) to node $n \in \mathcal{N}$. In this case, the preceding arcs are considered dummy arcs. This is done by introducing three sets of parameters W_{pnt}, V_{pnt}, and η_{ptmn} which reflect the effect of disruptions on the evacuation flow with respect to the arc incident times, sequence of arcs in the set of paths as well as topology of the network. Using these parameter a mathematical model with a linear structure can be developed. Note that a flow departing at time $t \in \mathbb{T}$ takes θ_{pn} time units to reach node $n \in \mathcal{N}$. Hence, based on the disturbed flow information $r_{pn(t-\theta_{pn})}$, we can address the flow reassignment from node $n \in \mathcal{N}$ onto route $p \in \wp$ during time interval $t - \theta_{pn} + \theta_{pn} = t$.

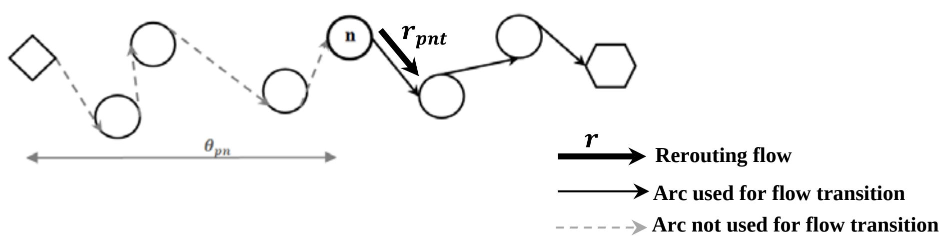

Figure 4. Representation of rerouting variable r_{pnt}

RPBM (Rerouting Path-Based Model) formulation:

We now describe the proposed dynamic network flow optimization model formulation. When an arc disruption occurs, disturbed evacuees are assumed to be accumulated on the tail of the affected arc (i.e., a node behind the affected road in the evacuation network) for the purpose of rerouting them to alternative paths. The objective function of *RPBM* aims to minimize the total number of disturbed evacuees remaining in the evacuation network by the end of the planning horizon T.

$$min \quad \sum_{n \in \mathcal{N}} hc_{nT}$$

Constraints are explained as follows. The planning horizon set $\mathbb{T} = \{0, 1, \dots, T\}$ covers both the pre-disruption schedule (f_{pt}) as well as the post-disruption schedule ($\mathcal{R}_{pnt}$). The time periods at which the previous plan is updated are shown by set $\mathbb{T}_{updated} = \{t_{updating}, \dots, T\}$. At time $t = 0$, there are no disturbed evacuees in the system. Hence, the total amount of associated flow on all nodes is set to zero as in Constraint (1).

$$hc_{n(t=0)} = 0, \qquad \forall\, n \in \mathcal{N}. \tag{1}$$

The process of a plan revision can only take place during the updating time interval ($\mathbb{T}_{updated}$). The flow-route assignments before $t_{updating}$ are equal to zero as in Constraint (2).

$$r_{pn(t-\acute{\theta}_{pn})} = 0, \qquad \forall p \in \wp, \forall a \in \mathcal{A}, n \in \mathcal{N}, t \in \mathbb{T}/\mathbb{T}_{updated}. \tag{2}$$

When calculating the amount of disturbed flow on node $n \in \mathcal{N}$ at time $t \in \mathbb{T}$, we take into account both the amount of the disturbed flow from the original plan (denoted by H_{pnt}) and the amount of the rerouted flow from nodes ($m \in \mathcal{N}$) that were disturbed while passing through the alternative pathway ($W_{pnt} \sum_{m\in\mathcal{N}} \eta_{ptmn} r_{pmt}$). This is stated in Constraint (3).

$$HC_{pn(t+\acute{\theta}_{pn})} = H_{pn(t+\acute{\theta}_{pn})} + W_{pnt} \sum_{m\in\mathcal{N}} \eta_{ptmn} r_{pmt}, \qquad \forall p \in \wp, n \in \mathcal{N}, t \in \mathbb{T}. \tag{3}$$

Parameter H_{pnt} used in Constraint (3) can be calculated as follows:

$$H_{pn(t+\acute{\theta}_{pn})} = \mathrm{V}_{pnt} f_{pt}, \qquad \forall p \in \wp, n \in \mathcal{N}, t \in \mathbb{T}.$$

When flow f_{pt} is blocked on node $n \in \mathcal{N}$ (denoted by $V_{pnt} = 1$), and since the flow has started from the origin of the path at time $t \in \mathbb{T}$, the time at which it reaches and accumulates on node $n \in \mathcal{N}$ is $t + \acute{\theta}_{pn}$. Note that $\acute{\theta}_{pn}$ is the transit time from the origin of path $p \in \wp$ to node $n \in \mathcal{N}$. Accordingly, $H_{pn(t+\acute{\theta}_{pn})}$ represents the number of evacuees disturbed on $n \in \mathcal{N}$ at time $(t + \acute{\theta}_{pn})$.

The total number of remaining interrupted evacuees at time $(t + 1)$ equals its previous amount at time $t \in \mathbb{T}$, plus the newly interrupted evacuees ($\sum_{p\in\wp} HC_{pn(t+1)}$), minus the amount of rerouted evacuees at time t, as expressed in the following equation.

$$hc_{n(t+1)} = hc_{\mathrm{nt}} + \sum_{p\in\wp} HC_{pn(t+1)} - \sum_{a\in\mathcal{A}} \sum_{p\in\wp} \Im_{pna} \left(1 - W_{pn(t-\theta_{pa})}\right) r_{pn(t-\theta_{pa})} \qquad \forall\, n \in \mathcal{N}, t \in \mathbb{T}. \tag{4}$$

Parameter $\Im_{pna}$ in Constraint (4) reflects the topology of the network and can be calculated as:

$$\Im_{pna} = \delta_{pa} \gamma_{na} \qquad \forall p \in \wp, a \in \mathcal{A}, n \in \mathcal{N}.$$

Considering constraints (2), (3), and (4), before the updating time, $hc_{n(t+1)}$ only equals the previous amount of the remaining flow plus the newly interrupted flow (i.e. $hc_{n(t+1)} = hc_{nt} + \sum_{p\in\wp} HC_{pn(t+1)}$). However, when the rerouting of the disturbed flow begins, the assigned disturbed flow $r_{pn(t-\theta_{pa})}$ at time $t \in \mathbb{T}$ to alternate paths is no longer stalled behind node $n \in \mathcal{N}$ and is subtracted from the remaining disturbed flow of the next time interval $hc_{n(t+1)}$. We use hc_{nt} as a measure to be used to later study the performance of our MIP model.

$$\sum_{p\in\mathcal{P}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}}\eta_{p(t-\theta_{pa})mn}\delta_{pa}\gamma_{na}\left[\left(1-V_{pn(t-\theta_{pa})}\right)L_{pm}f_{p(t-\theta_{pa})} + \left(1-W_{pn(t-\theta_{pa})}\right)\emptyset_{pmn}r_{pm(t-\theta_{pa})}\right]\leq C_a, \quad \forall a\in\mathcal{A},\ t\in\mathbb{T}. \quad (5)$$

Constraint (5) ensures that the total flow from different paths reaching a shared arc does not exceed the capacity of the arc (C_a). This flow includes (i) the pre-disruption flow schedule f_{pt}, and (ii) the post-disruption flow schedule r_{pnt}. Let us consider flow $f_{p(t-\theta_{pa})}$ departing from the origin of the path at time $t-\theta_{pa}$. Two cases can occur regarding the share of this flow in the capacity usage of arc $a\in\mathcal{A}$ (see Figure 5).

***Case 1**: Flow is disturbed before reaching arc $a\in\mathcal{A}$*

Case 1 occurs when there is at least one disruption on the preceding arcs before reaching arc $a\in\mathcal{A}$ and when the disruption time of an associated arc is less than the time required for the flow to reach and pass through the arc. Hence, if there is at least one arc $\bar{a}\in\mathcal{A}$ in which the following condition holds true,

$$\left(t-\theta_{p\bar{a}}\right)+\theta_{p\bar{a}}+\tau_{\bar{a}}>DT_{\bar{a}} \qquad \exists\bar{a}\in\mathcal{A} \text{ preceding to } a\in\mathcal{A} \text{ on } p\in\mathcal{P}$$

then, parameter $\eta_{p(t-\theta_{pa})mn}$ equals zero and $f_{p(t-\theta_{pa})}$ is not considered in the capacity Constraint (5). Note that $m\in\mathcal{N}$ represents the origin node of the path if $L_{pm}=1$. Also, $n\in\mathcal{N}$ represents the head of arc $a\in\mathcal{A}$ if $\gamma_{na}=1$. Note that the details of calculating the amount of parameter $\eta_{p(t-\theta_{pa})mn}$ using an algorithm is explained in Section 3.

***Case 2:** Flow can pass through arc $a\in\mathcal{A}$*

When the flow is not disturbed on preceding arcs to arc $a\in\mathcal{A}$ and reaches and passes through arc $a\in\mathcal{A}$, it occupies the capacity of arc $a\in\mathcal{A}$ until it completely passes through the arc. This occurs when the following condition holds:

$$\begin{aligned}\left(t-\theta_{p\bar{a}}\right)+\theta_{p\bar{a}}+\tau_{\bar{a}}&\leq DT_{\bar{a}}\\ \&\quad V_{pn(t-\theta_{pa})}&=0\end{aligned} \qquad \forall\bar{a}\in\mathcal{A} \text{ preceding to } a\in\mathcal{A} \text{ on } p\in\mathcal{P}$$

When the value of $\left(t-\theta_{p\bar{a}}\right)+\theta_{p\bar{a}}+\tau_{\bar{a}}$ is less than the disruption time ($DT_{\bar{a}}$) of a preceding arc $\bar{a}\in\mathcal{A}$, the flow can reach arc $a\in\mathcal{A}$, i.e., $\eta_{p(t-\theta_{pa})mn}=1$. At this point, if the flow is not disturbed on arc $a\in\mathcal{A}$ (i.e. $V_{pn(t-\theta_{pa})}=0$), it can flow through the arc, and the arc capacity is reduced accordingly. Note that Algorithm 2 explained in Section 3 is used to calculate the amount of parameter $\eta_{p(t-\theta_{pa})mn}$ to be used in the optimization model.

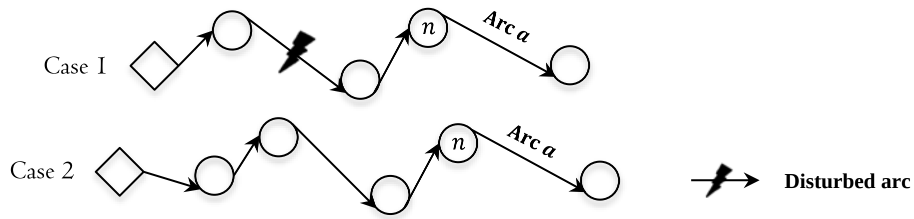

Figure 5. Case 1 and Case 2 presentation for flow $\boldsymbol{f}_{p(t-\theta_{pa})}$

Now, let us consider situations that may arise regarding post-disruption flow $r_{pm(t-\theta_{pa})}$(see Figure 6). Note that this flow is placed on path $p \in \wp$ through node $m \in \mathcal{N}$.

Case 1: *Reassigned flow has been added to the path from a preceding node to arc* $a \in \mathcal{A}$

When node $m \in \mathcal{N}$ *comes after arc* $a \in \mathcal{A}$ *on path* $p \in \wp$*, parameter* $\emptyset_{pmn}$ *equals zero and* $r_{pm(t-\theta_{pa})}$ *is excluded in the capacity constraint.*

Case 2: *Rerouted flow is interrupted before reaching arc* $a \in \mathcal{A}$

If node $m \in \mathcal{N}$ *is directionally placed before arc* $a \in \mathcal{A}$ *on path* $p \in \wp$ *(i.e.* $\emptyset_{pmn} = 1$*), but the rerouted flow is disrupted before arriving to the arc (*$\eta_{p(t-\theta_{pa})mn} = 0$*), then* $r_{pm(t-\theta_{pa})}$ *is disregarded in the constraint.*

Case 3: *Rerouted flow can travel through arc* $a \in \mathcal{A}$

If node $m \in \mathcal{N}$ *is directionally placed before arc* $a \in \mathcal{A}$ *on path* $p \in \wp$ *(i.e.* $\emptyset_{pmn} = 1$*), and the reassigned flow can reach the arc without any interruptions (*$\eta_{p(t-\theta_{pa})mn} = 1$*), then* $\mathcal{R}_{pm(t-\theta_{pa})}$ *is included in the arc capacity constraint.*

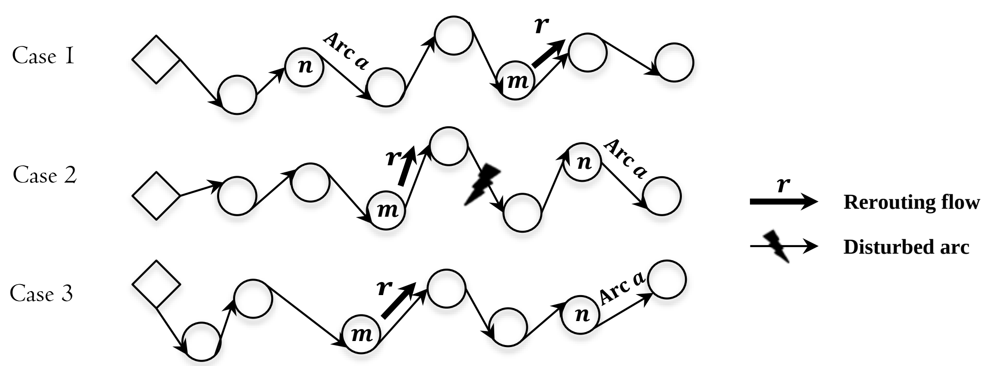

Figure 6. Case 1, Case 2, and Case 3 presentation for rerouted flow $r_{pm(t-\theta_{pa})}$

Constraint (6) ensures that all evacuees, including those who followed the pre-disruption plan and those that have been rerouted, are restricted by the capacity of the destination node (ℓ_n) when entering the shelter area.

$$\sum_{p\in\wp}\sum_{t\in\mathbb{T}} K_{pn}\left[\sum_{m\in\mathcal{N}_o}\eta_{ptmn}\, L_{pm}\, f_{pt}+\sum_{m\in\mathcal{N}}\eta_{ptmn}\, r_{pmt}\right]\leq \ell_n \qquad \forall n\in\mathcal{N}_d. \qquad (6)$$

Evacuation flow f_{pt} is considered in Constraint (6) only if the following condition holds:

$$\left(t-\theta_{p\bar{a}}\right)+\theta_{p\bar{a}}+\tau_{\bar{a}}\leq DT_{\bar{a}} \qquad \forall \bar{a}\in\mathcal{A} \textit{ between the source node and destination node of path } p\in\wp$$

Hence, starting from the origin and moving towards the destination, if the flow of an evacuee is not interrupted ($\eta_{ptmn} = 1$ where $m \in \mathcal{N}_o$ and $n \in \mathcal{N}_d$), it is counted in Constraint (6). Similarly, the rerouted flow r_{pmt} is considered to use the capacity of the destination node only if no incident affects the flow from node m $\in \mathcal{N}$ (the point it had been inserted on the path) to the destination node $n \in \mathcal{N}_d$ ($\eta_{ptmn} = 1$).

Finally, Constraints (7) reflects the non-negativity and integrality of decision variables, respectively.

$$r_{pnt}\in\mathbb{Z}^+, hc_{nt}\in\mathbb{Z}^+, \qquad \forall p\in\wp, n\in\mathcal{N}, t\in\mathbb{T}. \qquad (7)$$

3 Algorithms to Calculate Key Model Parameters and Network Clearance Time

As explained in Appendix B, developing an optimization model for the problem that can be solved in a timely manner was challenging. However, upon examining the problem characteristics, we realized that a mixed integer linear program can be developed if some key parameter values are determined *a priori*. Hence, this section introduces two preprocessing algorithms to calculate the parameter values, which makes it possible to solve the proposed $RPBM$ model.

3.1 Methodology to Calculate Parameters Associated with Disruptions

Parameter V_{pnt} indicates whether or not flow f_{pt} on path $p \in \wp$ departing at time $t \in \mathbb{T}$ would be stalled on node $n \in \mathcal{N}$. *Algorithm 1* is developed to calculate the value of V_{pnt}. First, we calculate W_{pnt} to define whether f_{pt} is stopped on node $n \in \mathcal{N}$ regardless of any possible disturbances that may have surfaced during the flow passage up to node $n \in \mathcal{N}$. Accordingly, we initialize disruption times of preceding arcs to node $n \in \mathcal{N}$ to be infinity. We disregard disturbances of the preceding arcs and only take into account the time of incident (DT_a) of the arc that emerges from node $n \in \mathcal{N}$, arc transit time (τ_a), and the time to transport from the origin of the route to arc $a \in \mathcal{A}$ (θ_{pa}). If a flow departs on path $p \in \wp$ at time $t \in \mathbb{T}$, it reaches arc $a \in \mathcal{A}$ at time $t + \theta_{pa}$. Also, it takes τ_a unit of time for the flow to completely pass through the arc. Therefore, if $t + \theta_{pa} + \tau_a$ is greater than the disruption time of the arc (DT_a), the flow is interrupted, and W_{pnt} takes value 1.

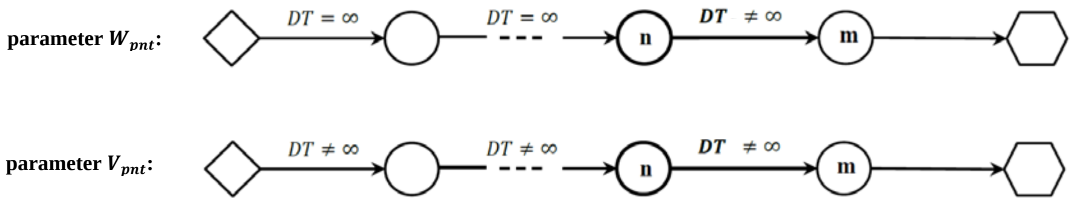

Figure 7: Assumptions used to define parameters W_{pnt} and v_{pnt}

Next, we calculate the value of V_{pnt} and subsequently take into account incident times of preceding arcs to arc $a \in \mathcal{A}$. The value of V_{pnt} equals 1 only if the flow f_{pt} experiences no interruption while moving toward node $n \in \mathcal{N}$ (i.e., $\sum_{m \in \mathcal{N}} W_{pmt} = 0$) and is disturbed on node $n \in \mathcal{N}$ of path $p \in \wp$ (i.e., $W_{pnt} = 1$). Else $V_{pnt} = 0$.

Algorithm 1

Inputs:

An evacuation network $\mathcal{G}$ consisting of a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$.
Disruption Time of Arcs

Calculating V_{pnt}:

```
for all paths p ∈ ℘ do
    for all time slots t ∈ 𝕋 do
        for all arcs that belong to path p ∈ ℘ do
            if t + θ_pa + τ_a − DT_a > 0 then
                W_pnt = 1 (n is upstream node of arc a)
            else if t + θ_pa + τ_a − DT_a ≤ 0 then
                W_pnt = 0
            end if
            for all preceding nodes m ∈ 𝒩 to arc a on path p do
                if ∑_{m∈𝒩} W_pmt = 0 and W_pnt = 1 then
                    V_pnt = 1 (n is upstream node of arc a)
                else then
                    V_pnt = 0
                end if
            end for
        end for
    end for
end for
```

A numerical example for calculating the value of V_{pnt}:

The following example is used to illustrate the calculation of parameter V_{pnt}. Consider the path, disruption times, and arc transit times shown in Figure 8. When an evacuee departs from node i at time $t = 1$, the evacuee can pass through arc (i,j) because the disruption on arc $DT_{(i,j)} = 4$ occurs after the flow has reached node j (i.e., time $t = 2$). This evacuee can also pass through arc (j,k) with no interruption, as the time it arrives at node k (i.e., time $t = 2 + 5 = 7$) is earlier than the time of the incident on arc $DT_{(j,k)} = 8$. Accordingly, this flow is not disturbed on either of the arcs (i,j) or (j,k) and $V_{pi(t=1)} = V_{pj(t=1)} = 0$. Now, let us consider a flow starting on the path at time $t = 3$. It will arrive at the origin of the arc (j, k) at time $t = 4$ with no disturbance. The arc transit time for arc (j, k) is 5 units of time. Thus, it cannot pass through the arc because the incident occurred prior to the projected arrival to the location, *i.e.*, $4 + 5 > 8$. Associated values of V_{pnt} will be $V_{pi(t=3)} = 0$ and $V_{pj(t=3)} = 1$, as the disturbed flow is only affected on node j. Similarly, for the departure time $t = 5$, we would have $V_{pi(t=5)} = 1$ because the flow was not affected as the incident occurred prior to the departure time.

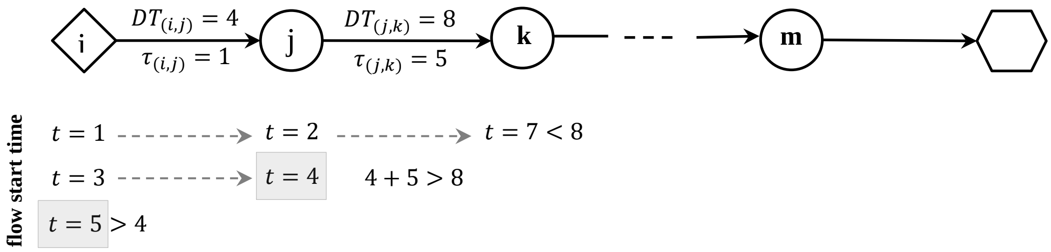

Figure 8: Example for parameter v_{pnt}

Next, *Algorithm 2* is developed to determine whether a flow can pass through a specific location in the network and can reach another location without any interruptions. For any path $p \in \mathcal{p}$, we first derive the sequence of nodes composing the path called φ_p. Then, for any combination of node $m \in \mathcal{N}$ and $n \in \mathcal{N}$ in the set φ_p (when m is a precedence to node n), we calculate the summation of $\sum_{k=m}^{n-1} V_{pkt}$. If $\sum_{k=m}^{n-1} V_{pkt} = 0$, we can conclude that the flow on path $p \in \mathcal{p}$ at time $t \in \mathbb{T}$ is neither interrupted on node $m \in \mathcal{N}$ nor is disturbed between node $m \in \mathcal{N}$ and node $n \in \mathcal{N}$. If this condition holds, then $\eta_{ptmn} = 1$. Otherwise, $\eta_{ptmn} = 0$.

Algorithm 2

Inputs:

An evacuation network $\mathcal{G}$ consisting of a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$, and v_{pnt}.

Calculating $\boldsymbol{\eta_{ptmn}}$:

```
for all paths p ∈ 𝓅 do
  determine set φ_p as sequence of nodes of path p
    for all time slots t ∈ 𝕋 do
      for all preceding nodes m ∈ φ_p to node n ∈ φ_p do
        if Σ_{k=m}^{n-1} v_pkt = 0 then
          η_ptmn = 1
        else then
          η_ptmn = 0
        end if
      end for
    end for
end for
```

3.2 Rerouted Clearance Time Calculation

This section explains the procedure to calculate a performance measure *rerouted clearance time*

(RCT), which is defined as the minimum time to safely evacuate people from harm's way to safe destinations considering the route adjustments during the evacuation. One way to calculate RCT is to solve the $RPBM$ model in sequence of a planning horizon $\mathbb{T} = \{0, 1, \dots, T\}$. The smallest value in $\mathbb{T}$ whose objective value is zero is the RCT for the problem. However, such a trial-and-error approach is tedious and time consuming. Hence, we introduce a two-step procedure to expedite the process of finding the RCT value. The first step is to quickly approximate the RCT for $RPBM$. Using this value as a starting solution, *Algorithm 3* finds the exact value of RCT as the final step. Let $\mathfrak{D}_{nt} = \sum_{p\in\wp}\sum_{m\in\mathcal{N}} L_{pn}\, H_{pmt}$ denote the demand (evacuees) on source node $n \in \mathcal{N}_o$ of path $p \in \wp$ at time $t \in \mathbb{T}$. We aim to reroute these evacuees through the marginal (residual) capacity of the roads and find the shortest amount of time to reach safe shelters. The residual capacity of the arcs (Ꝛ_{at}) and the destination nodes (ƀ_n) are calculated as follows:

$$\text{Ꝛ}_{at} = C_a - \sum_{p\in\wp}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}} \eta_{p(t-\theta_{pa})mn}\Im_{pna}\left(1 - V_{pn(t-\theta_{pa})}\right) L_{pm} f_{p(t-\theta_{pa})} \qquad \forall a \in \mathcal{A},\ t \in \mathbb{T}\ \&\ t < DT_a, \tag{8}$$

$$\text{ƀ}_n = \sum_{p\in\wp}\sum_{t\in\mathbb{T}}\sum_{m\in\mathcal{N}_o} K_{pn}\eta_{ptmn}\, L_{pm}\, f_{pt} \qquad \forall n \in \mathcal{N}_d, \tag{9}$$

The solution of the following optimization model gives an approximated RCT.

$$Min \quad Max_{p\in\wp, t\in\mathbb{T}}\left(\mathfrak{u}_{pt}(t + d_p)\right)$$

S.t.

$$\sum_{p\in\wp}\sum_{t\in\mathbb{T}} L_{pn}\, \text{ꞙ}_{pt} \geq \sum_{t\in\mathbb{T}} \mathfrak{D}_{nt}, \qquad \forall n \in \mathcal{N}_o, \tag{10}$$

$$\sum_{p\in\wp} \partial_{pn} L_{pn}\delta_{pa}\text{ꞙ}_{p(t-\theta_{pa})} \leq \text{Ꝛ}_{at}, \qquad \forall a \in \mathcal{A},\ \forall t \in \mathbb{T}, n \in \mathcal{N} \tag{11}$$

$$\sum_{p\in\wp}\sum_{t\in\mathbb{T}} K_{pn}\, \text{ꞙ}_{pt} \leq \text{ƀ}_n, \qquad \forall n \in \mathcal{N}_d, \tag{12}$$

$$M\, \mathfrak{u}_{pt} \geq \text{ꞙ}_{pt} \qquad \forall p \in \wp, \forall t \in \mathbb{T}, \tag{13}$$

$$\text{ꞙ}_{pt} \in \mathbb{Z}^+, \mathfrak{u}_{pt} \in \{0,1\} \qquad \forall p \in \wp, \forall t \in \mathbb{T}, \tag{14}$$

Variable ꞙ_{pt} is the amount of flow from the source node of path $p \in \wp$ at time $t \in \mathbb{T}$. The objective function aims to minimize the maximum time in which a flow reaches its destination. Constraints (10)-(14) are used to reroute $\mathfrak{D}_{nt}$ from source nodes $\mathcal{N}_o$ through the residual capacity of the network. According to Constraint (13), binary variable $\mathfrak{u}_{pt}$ receives value 1 if the flow is reassigned to path $p \in \wp$ at time $t \in \mathbb{T}$. For $\mathfrak{u}_{pt} = 1$, the associated flow reaches the destination d_p unit of time later during time interval $(t + d_p)$, where d_p is the duration time of the path.

Let $\mathfrak{X}$ represent the solution of the above optimization model. Time value of $\mathfrak{X}$ is used in *Algorithm 3* to expedite the RCT calculation process. The algorithm has two main steps: RCT bound generation and value calculation. In bound generation, if $\sum_{n\in\mathcal{N}}\sum_{t\in\mathbb{T}} I_{nt} > 0$, it means there still remains some disturbed evacuees in the network, and the system is not cleared; hence, it provides a lower bound to RCT. If $\sum_{n\in\mathcal{N}}\sum_{t\in\mathbb{T}} I_{nt} \leq 0$, it means the system has been cleared. However, we are aiming to find the minimum amount of time required to clear the system; hence, we consider $\mathfrak{X}$ as a lower bound to RCT. In the second part of the algorithm to calculate RCT, the RPBM is solved considering T and $T-1$ as the planning horizon, where $T = (\mathrm{LB_{RCT}} + \mathrm{UB_{RCT}})/2$. If the objective value at $\mathrm{T}-1$ is positive (i.e., the system is not cleared) and becomes zero at time T (i.e., the system is cleared), the minimum time to clear the network is achieved at T; $RCT = T$. If both objective values are positive, the system cannot be cleared at time T. Hence, we update the lower bound of RCT to $LB_{RCT} = T$ if $T > LB_{RCT}$. If both objective values equal zero, it means that the system has been cleared at $\mathrm{T}-1$. This triggers updating the upper bound to $UB_{RCT} = T - 1$ if $T - 1 < UB_{RCT}$. The algorithm continues until RCT value is found.

Algorithm 3: Calculating Clearance time of the reroute plan

```
Inputs:
  RPBM and 𝔛.
  put CT of the evacuation network with no disruption as LB_RCT
  put UB_RCT = 2 LB_RCT
Calculating a tight bound for RCT:
  put 𝕋 = {0, 1, … , 𝔛} as the planning horizon
  solve RPBM
  if ∑_{n∈𝒩} ∑_{t∈𝕋} I_nt > 0 then
        LB_RCT = 𝔛
  else then
        UB_RCT = 𝔛
  end if
Calculating RCT Value:
  put 𝕋 = {0, 1, … , T} as the planning horizon
  While ψ ≠ 1 do
        put T = (LB_RCT + UB_RCT)/2
        Solve RPBM for 𝕋 = {0, 1, … , T} and 𝕋 = {0, 1, … , T − 1}
        if one objective value is a positive value and the other is equal to zero then
              put RCT = T and ψ = 1
        else if both objective values are positive and T > LB_RCT then
              put LB_RCT = T
        else if both objective values are equal to zero and T − 1 < UB_RCT then
              put UB_RCT = T − 1
        end if
  end while
```

4 Computational Results

The computational experiments presented in this study focus on the impact of network arc disruptions on evacuation plans and the performance of the proposed recovery strategy. First, the performance of $RPBM$ is investigated on a small sample network in generating alternative routes. Then, the same experiment is conducted on a real evacuation network involving a large metropolitan area. The optimization models are solved using CPLEX 12.5.1, while experiments are performed on a PC with a 3.07 GHz Intel Core i7 processor having 24GB RAM and running Ubuntu 10.04.3.

4.1 Numerical experiments to illustrate the proposed approach

Experimental studies are conducted on the sample network shown in Figure 9 (Lim et al., 2012). The test network includes three source nodes ($\mathcal{N}_1$, $\mathcal{N}_2$, and $\mathcal{N}_3$), five intermediate nodes ($\mathcal{N}_4$,..., $\mathcal{N}_8$) and two destination nodes ($\mathcal{N}_9$ and $\mathcal{N}_{10}$). These nodes are connected through 22 arcs. Arc transit times (τ_a) as well as arc capacities (C_a) are shown above each arc of the network.

Demand on the source nodes (number of evacuees that are present at the source nodes) are assumed to be $D_1 = 110$, $D_2 = 120$, and $D_3 = 167$, and the capacity of each of the destination nodes is assumed to be 750. First, all possible paths between all origin and destination (O-D) pairs are enumerated using the solution pool feature of *CPLEX* for the shortest path problem. Next, a total of 42 shortest paths are selected as the candidate paths with 13 paths originating from source node $\mathcal{N}_1$, while 14 paths (P_{14}-P_{27}) originate from the second node, and 15 paths (P_{28}-P_{42}) originate from the third node as listed in Table 7 in Appendix B. For instance, P_{12} follows the sequence of $\mathcal{N}_1 \rightarrow \mathcal{N}_5 \rightarrow \mathcal{N}_6 \rightarrow \mathcal{N}_8 \rightarrow \mathcal{N}_{10}$. These candidate paths are used as input data to the path-based evacuation model (see Appendix A) for the purpose of generating an initial evacuation plan.

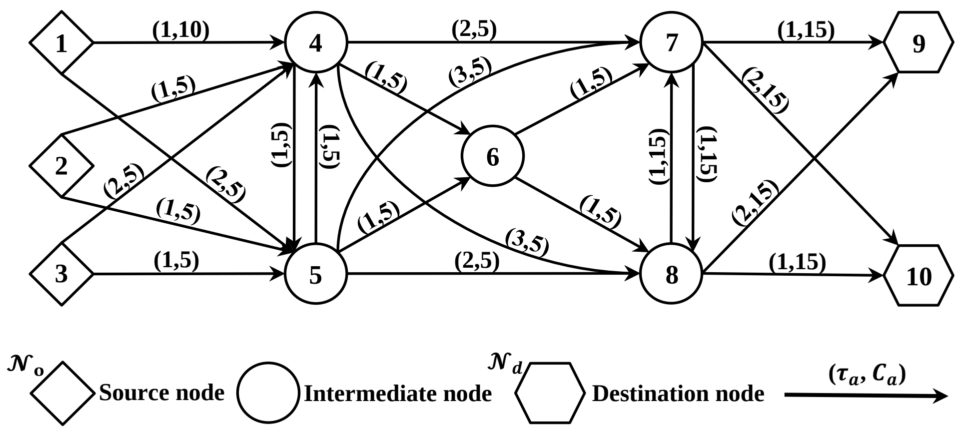

Figure 9: Evacuation test network

The path-based evacuation optimization model provides both the optimal pre-disruption route assignment and flow schedule f_{pt} for the test case as in Table 1. For example, as shown in the first row of the Table, five evacuees ($f_{P9,t2} = 5$) commence the evacuation by leaving node 1 following path P_9 at time $t = 2$. The evacuation rate varies during time $t = 3$, $t = 4$, and $t = 8$: $f_{P9,t3} = 2$, $f_{P9,t4} = 5$, and $f_{P9,t8} = 5$. The corresponding clearance time is 22 time units.

Suppose that three incidents occurred on different roads during the evacuation. The disturbed roads (i.e., arcs) are labeled as $A^{disturbed} = \{(2,5), (4,5), (4,7)\}$. Incidents are assumed to occur at different times: $DT_{a(2,5)} = 4$, $DT_{a(4,5)} = 6$, and $DT_{a(4,7)} = 8$. While monitoring the progress of the disaster, emergency management agencies received data on the network condition and attempted to develop revised plans accordingly. As the process of collecting information and rerouting plan generation takes time, it is assumed that agencies can implement the revised schedule immediately after the plan updating time

$t_{updating} = 10$. Hence, before $t = 10$, no rerouting itinerary is planned and the corresponding variable ($\mathcal{R}_{pnt}$) remains at zero.

Table 1. An initial evacuation plan for the sample network (f_{pt})

Paths	Time Slots 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
P9		5	2	5				5										
P10	5	5		5		5	5	5	5	5	5	5		5		5	5	
P11	5						5	3		5	5					5		
P16																		5
P17													5	5				
P18						5												
P19		2		5												5		
P21		3																
P22					5		5				5	5	5			5	5	
P23		2	2	5				5	5	5				5	5			
P24	5		3			5												
P27		3																
P28																		5
P30		3																
P31	3					5				5					5	5	5	
P33				5							5	5						
P35															5			
P36						5		5	5	5			5	5		5		
P37		2	5	5	5		5					5					5	
P39	5																	
P40											5							
P41	2	5	5		5		2	5					5					
P42														5				

Disruptions on arcs (2,5), (4,5), and (4,7) partially affect several paths $\{P_{17}, P_{18}, P_{19}, P_{21}, P_{22}, P_{23}, P_{27}, P_{31}, P_{41}, P_{42}\}$. For example, flow $f_{p23,t2} = 2$ was scheduled to arrive at arc (4,7) at time $t + \theta_{p23,a(4,7)} = 2 + 1 = 3$. It takes $\tau_{a(4,7)} = 2$ time units for the flow to pass through this arc. Since the arc is supposed to fail at time $DT_{a(4,7)} = 8$, the flow $f_{p23,t2} = 2$ is not affected. Now, flow $f_{p23,t9} = 5$ is scheduled to reach arc (4,7) at time $t + \theta_{p23,a(4,7)} = 9 + 1 = 10$, but the arc is already blocked at time $t = 8$. So, there will be a flow accumulation on node 2 at time $t = 10$, and it is denoted by $H_{p23,n4,t10} = 5$. The magnitude, location, and interruption time of the disturbed flow H_{pnt} are demonstrated in Table 2.

Table 2. Disturbed flow (H_{pnt})

Paths		Time Slots 4	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total
P17	*n2*									5	5						
P18	*n2*		5														**25**
P19	*n2*	5											5				
P22	*n4*		5		5				5	5	5			5	5		
P23	*n4*					5	5	5				5	5				
P31	*n4*				5				5					5	5	5	**107**
P41	*n4*			5		2	5					5					
P42	*n4*												5				

According to Table 2, 132 evacuees out of 392 are constrained at different locations of the transportation network between time periods $t = 4$ and $t = 19$. Hence, the proposed RPBM model is used to generate new paths to accommodate the disturbed flow.

Table 3 shows the rerouting schedule for the disturbed flow r_{pnt} provided by $RPBM$. The first row of the table shows the flow that should be reassigned to P_{21} from node 2. The remaining rows show route assignments from node 4 onto alternative paths $\{P_2,\ P_8, P_9, P_{10}, P_{15}, P_{16}, P_{25}, P_{26}, P_{33}\}$. This plan was able to reroute all interrupted flow and redirect them to safe shelters within 35 time intervals; hence, the corresponding RCT is 35 time units. During this time period, a total of 25 evacuees are evacuated from node 2, and 132 evacuees are rerouted from node 4.

Table 3. Rerouting plan (r_{pnt})

		Time Slots																Total
		13	15	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
P21	**n2**								5	5		5	5			5		**25**
P2											2							
P8												5						
P9																5		
P10		5		5	5	5	5					5		5		5		
P15	**n4**												5	5			5	**132**
P16								5										
P25					5	5	5		5									
P26			5					5	5	5	5		5		5			
P33									5									

Earlier, Table 2 showed that only 107 evacuees experience congestion on node 4 as a result of the incidents. However, the plan provided by $RPBM$ assigns 132 evacuees from this node onto new pathways. This is due to the fact that, in $RPBM$, disrupted flow could still reach disrupted arcs after its reroute assignment and thus add to the accumulated flow behind the failed arcs. Consequently, the overall amount of disturbances calculated in $RPBM$ shown in Table 4 could be higher than that of the associated amount shown in Table 2.

Table 4. Total disturbed flow from the initial and the rerouted plan (HC_{pnt})

			Time Slots																				
			4	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*24*	*25*	*27*	*28*	*31*	***Total***
Paths	***P17***	***n2***									5	5											
	P18	***n2***		5																			**25**
	P19	***n2***	5											5									
	P21	***n4***																5	5	5	5	5	
	P22	***n4***		5		5				5	5	5			5	5							
	P23	***n4***					5	5	5				5	5									
	P31	***n4***				5				5					5	5	5						**132**
	P41	***n4***			5		2	5					5										
	P42	***n4***												5									

According to Table 4, 132 disturbed evacuees are associated with the pre-disruption flow ($H_{pn(t+\hat{\theta}_{pn})}$), and 25 disturbed evacuees highlighted in the table (from time $t = 24$ to $t = 31$) belong to the disrupted rerouted flow ($W_{pnt} \sum_{m \in \mathcal{N}} \eta_{ptmn} r_{pmt}$). The disruptions on the rerouted flow happened on Path 21 and created traffic congestion behind Node 4 during time slots 24, 25, 27, 28, and 31. As shown in Table 3, these disturbed flows have been previously rerouted onto Path 21 (through node 2) at time slots 23, 24, 26, 27, 30. Note that the node sequence of Path 21 is *2-4-5-6-8-10*. Hence, the starting flow from Node 2 reached Node 4 within one unit of time. Since it could not pass through arc (4,5), the flow is stalled behind Node 4 one time unit later at times 24, 25, 27, 28, and 31.

The performance of the proposed model is further investigated using four different test instances. Table 5 shows the input data for these instances and includes information regarding the set of disrupted arcs, corresponding disruption times, and updating times for the rerouting strategy.

Table 5. Test problems

	C1	C2	C3	C4
Disrupted arcs	(2,5),(4,5), (4,7),(6,7)	(5,4),(5,7), (6,8),(7,10)	(2,5),(4,5), (5,7),(8,7)	(2,5),(4,5),(4,6), (5,6),(8,7),(7,10)
Disruption time	{9,16,14,8}	{17,8,10,19}	{12,15,14,7}	{15,6,8,10,9,12}
Updating time	16	19	15	15

Figures 10 highlights the amount of remaining disturbed flow (hc_{nt}) in the system as the evacuation progresses. The remaining disturbed flow gradually increased at early stages of the planning horizon. When the rerouting process began, hc_{nt} gradually decreased until there remains no disturbed flow to be rerouted. The system was cleared when all evacuees reached the destination nodes. In test problem C_1, the remaining disturbed flow was zero at time $t = 32$. But, it took additional 2 units of time for the last rerouted flow to reach the destination via the assigned alternative path. Hence, the system was cleared at time 34, i.e., RCT=34. Similarly, the RCT for test problems C_2 , C_3, and C_4 are 32, 25, and 29, respectively.

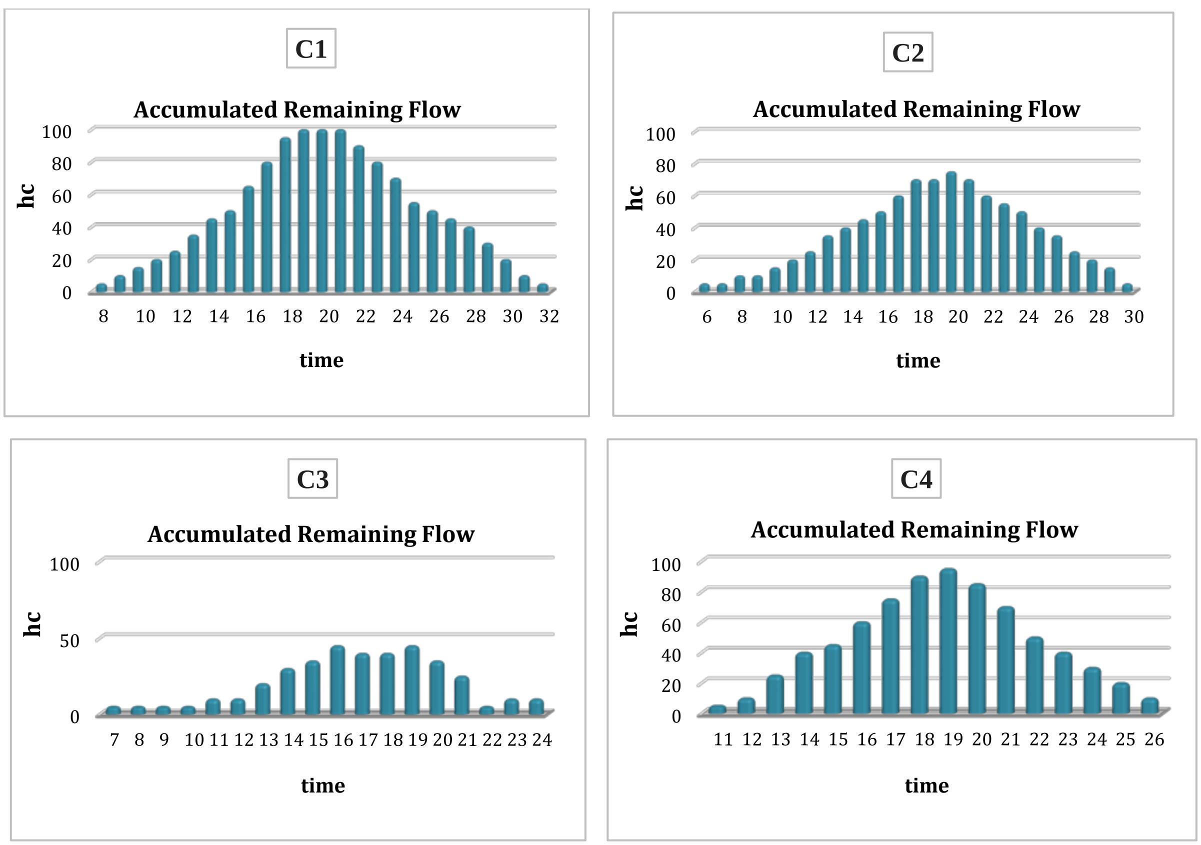

Figure 10. Accumulated remaining flow (hc_{pnt}) under different sample problems

The rerouted clearance times are different for each problem instance because the input data for the four cases are different in terms of the disrupted arcs, the arc disruption times, and the updating times. All of these factors have influence on the amount of clearance time of the network. In the following, we provide a sensitivity analysis on the evacuation plans under different problem settings for these factors.

Effect of the network topology (disruption on arcs):

When an incident happens in the evacuation network, the time of completion of the rerouting process can be impacted depending on different factor such as the location of the road in the network, its level of connectivity to other roads, the residual (remaining) capacity of the adjacent roads, and the proximity of the incident location to either the source nodes or the destination nodes. Hence, this section analyzes the effect of disruption on each arc to determine which arc makes the network evacuation plan the most vulnerable by fixing the values of the other two factors. The clearance times of the rerouting plans, accounting for the disruption of each of the 22 arcs, are shown in Figure 11.

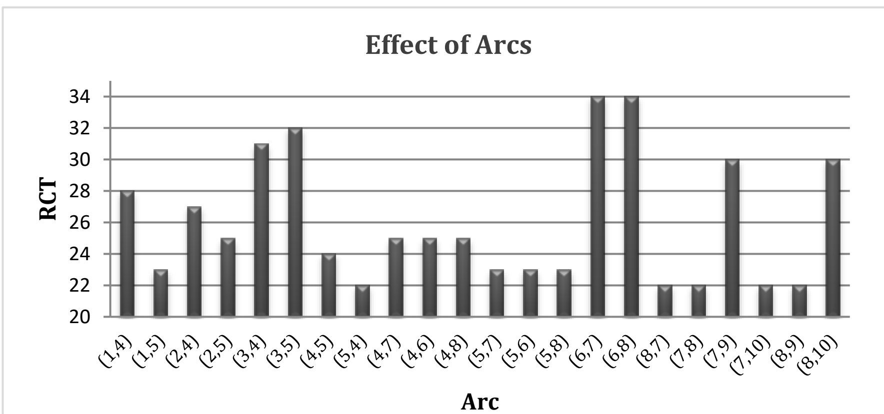

Figure 11. Effect of arc disruption on evacuation process

As shown, the most vulnerable arcs affecting the pre-disruption plan are (6,7) and (6,8). This is due to the fact that for these two arcs, it takes the longest time (RCT =34 time units) to reroute the disturbed flow and clear the network following their disruptions. The second most vulnerable arcs are (3,4), (3,5), (7,9) and (8,10) with a corresponding RCT ranging from 30 to 32. Finally, the least vulnerable arcs are (5,4), (8,7), (7,8), (7,10), and (8,9). Disruption on these arcs disturbs part of the flow; However, these disruptions do not change the RCT of the plan. The rerouting plan was able to use the residual capacity of the evacuation network to accommodate the disturbed flow onto alternate paths and redirect them to safe shelters within the same clearance time of the pre-disruption plan (CT=22 units of time). The amount of disturbed flow, the time range of flow disturbance as well as the rerouting time intervals, are shown in Table 6.

Table 6. Analysis of Less Vulnerable Arcs

Arc	(5,4)	(8,7)	(7,8)	(7,10)	(8,9)
Total disrupted	35	10	30	0	0
Flow disruption time	[9,17]	[11,14]	[9,18]	0	0
Arc capacity	5	15	15	15	15
Rerouting node	n5	n8	n7	-	-
Rerouting time	[11,17]	[11,17]	[11,20]	0	0

Note that the amount of disrupted flow is equal to the amount of rerouted flow. The rerouting node represents the upstream node of the disrupted arc from which the disturbed flow is rerouted. When arc (5,4) experiences a disturbance at time 9, it causes the disruption of 35 evacuees during the time intervals between 9 and 17. This amount of flow is gathered behind Node 5 and, consequently, is rerouted from the same node between time 11 and 17. Hence, the rerouting process ends before time 22, which also happens to be the CT of the pre-disruption plan. Cases for arcs (8,7) and (7,8) are similar. However, disruptions of arc (7,10) and (8,9) have no effect on the pre-disruption plan and no evacuees are disturbed. This is because the affected arcs are not associated with the paths used in the pre-disruption plan.

Effect of disruption times:

The occurrence time of an incident can be an important factor influencing the rerouted clearance time. For instance, if an incident occurs on an arc, it will create a disruption only to evacuees who have not yet passed the incident location in the route. Hence, this section studies the effect of arc disruption times on the evacuation and rerouting process. For this purpose, a set of disturbed arcs are chosen as $A^{disturbed} = \{(2,4),(4,8)\}$. The disruption times of the arcs are changed in the interval [1, 21], and an update will be triggered one unit after arc disruption times. Figure 12 (a) shows the RCTs when the disruption time of the arcs changes between time 1 and time 21. As can be seen, the rerouting CT is higher when the disruption time occurs earlier in the planning horizon. This is not surprising because as a disruption happens earlier on the road, more evacuees are affected, and more time is required to reroute them and clear the system.

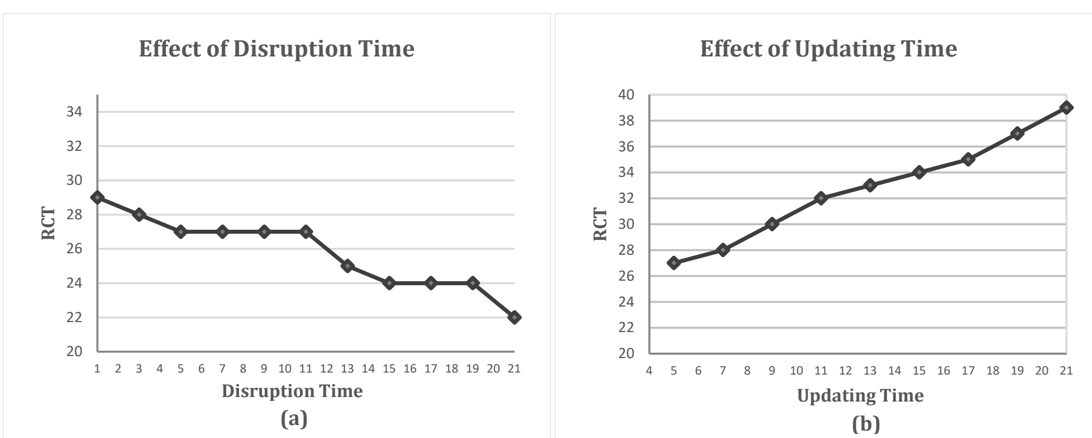

Figure 12. Effect of disruption time and updating time on evacuation process

Effect of information (updating time):

Shortly after an incident occurs within the network, it starts to disrupt the evacuation flow. It takes time for the planners to (1) understand and analyze the situation, (2) make an appropriate decision on the rerouting strategy, and (3) execute the reroute plan accordingly. To account for this delay in reroute planning, we introduce an updating time $t_{updating}$ to capture the total time it took from the incidence analysis until the new plan is ordered to be executed. The effect of $t_{updating}$ on the evacuation process is studied and is shown in Figure 12 (b). The more delay there is in receiving update information on the network situation, the longer the clearance time of the network after disturbed flow reassignments. Note that it took less than a second for running each preprocessing algorithm as well as the MIP model on the small network of Figure 8.

4.2 Numerical Experiments on a Large-Scale Network

We continue the experiments on an evacuation network of the Greater Houston area (Lim et al., 2012). Houston, Texas, the fourth largest city in the U.S., is known to be one of the most vulnerable metropolitan cities situated on the Gulf Coast, and has been severely affected by hurricanes and floods for the several decades. The Houston network (Figure 13) comprises of a total of 42 nodes and 107 arcs. The first thirteen nodes represent source nodes ($\mathcal{N}_1$ -$\mathcal{N}_{13}$), and the last four nodes ($\mathcal{N}_{39}$ -$\mathcal{N}_{42}$) represent safe destination nodes.

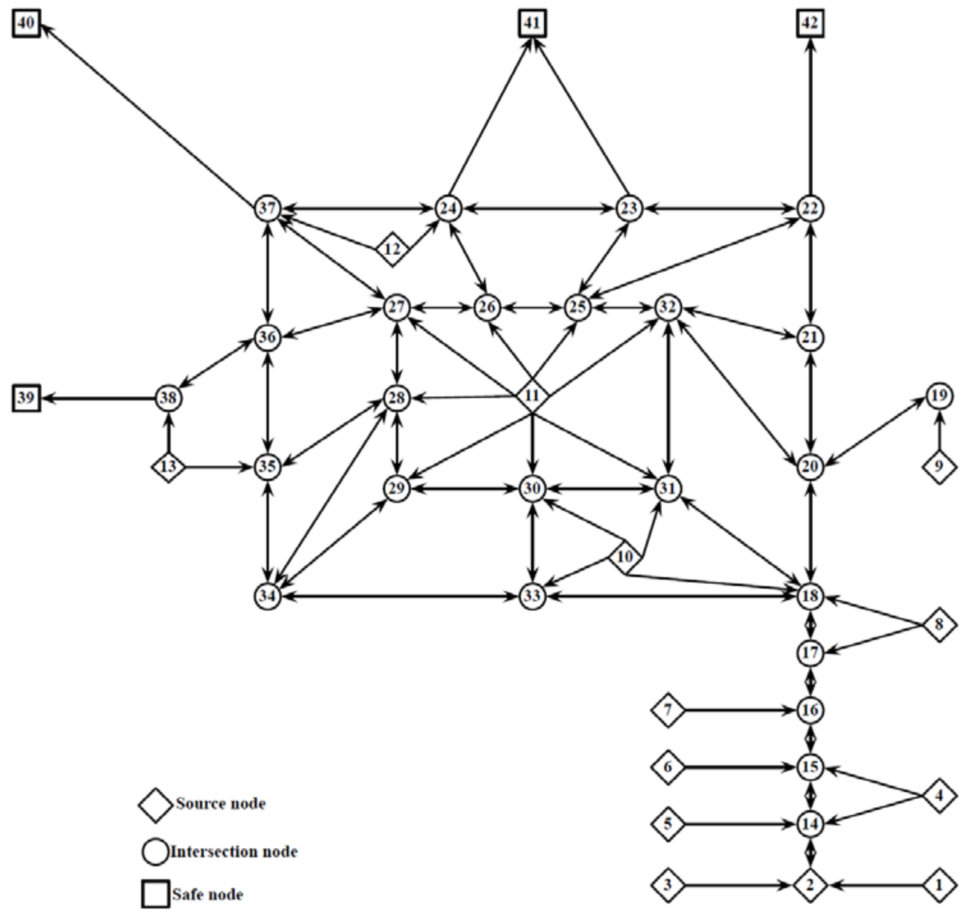

Figure 13. City of Houston transportation network

For the purpose of demonstrating our proposed evacuation rerouting approach, we used the same input data for this network as it was reported in Lim et al., 2012. Hence, the total number of evacuees (i.e., evacuation vehicles) on the source nodes are assumed to be 56,600, in which each of source nodes 1-6 has 100 evacuees, 3,500 for each of nodes 7-10, and 14,000 evacuees each for nodes 11-13. The transit times are defined to be multiplies of $\tau = 30$ minute intervals. Using the PBM model in Appendix A, we first generate the pre-disruption evacuation plan using 140 candidate paths to be selected in the optimization model. The resulting pre-disruption plan distributed the evacuation flow over 52 selected paths, and it took 129τ to clear the network. Disruptions are triggered on arcs (22,42), (20,32), (11,27), and (35,34) at times 56, 69, 67 and 48, respectively. These road disruptions affect flows on 11 paths and result in 10,966 evacuees being stranded behind nodes 11, 20, 22, and 35. Among these evacuees, 600 are on node 11, 1,675 on node 20, 7,331 on node 22, and 1,360 on node 35. The $RPBM$ is used to provide a reroute plan for the disturbed flow. The total combined computation time of running both *Algorithm 1* and *Algorithm 2* was 67.91 seconds, while it took 203.60 seconds to run *Algorithm 3* to calculate RCT. It took 52.41 seconds to solve the $RPBM$ based on the RCT value. The corresponding reroute plan is shown in Figure 14. In the figure, the amount of rerouted flow from each node during different time intervals are illustrated. The total time taken to move all disturbed flow to safe shelters was 162τ. This means that our model required approximately 16 hours to adjust the plan and rearrange 251,850 disturbed evacuees to safe shelters through the residual capacity of the Houston network after disruption.

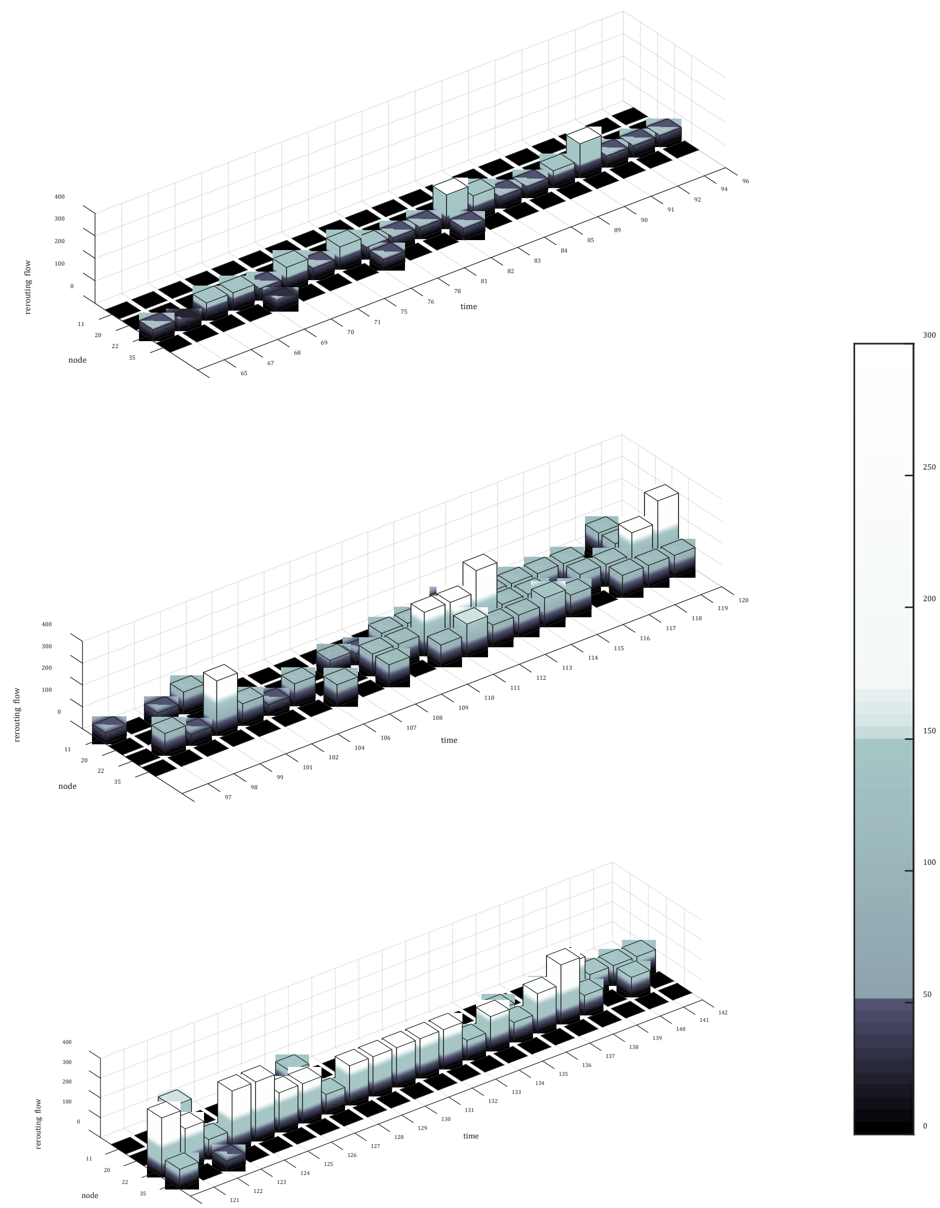

Figure 14. Rerouting Evacuation plan for Houston transportation network

5 Conclusion

This paper introduced a real-time rerouting evacuation strategy that can be applied to post disruption networks to minimize evacuation clearance time in response to the occurrence of real-time incidents. For this purpose, a dynamic network flow optimization model formulation ($RPBM$) was introduced, in which variable evacuation flow rates are considered to develop alternative paths to achieve more practical and effective evacuation plans. Due to very high-level complexity for developing a practically useful optimization model, computational algorithms have been developed to calculate a few key values for specific parameters related to road disruption. As a result, it enabled us to develop a simple and computational efficient MIP model for the evacuation problem. A rerouting clearance time calculation algorithm is introduced to efficiently calculate the minimum amount of time required to mobilize disturbed evacuees to the safe shelters. Numerical experiments were thoroughly conducted to study the performance of the proposed $RPBM$ under different problem configurations. Computational experiments were made to test computational efficient in solving the proposed approach. The impact of three incident-related factors have been investigated to better understand their effects on the rerouting process, including the location of the disruption, the time of disruption occurrence, and the plan updating time. The results showed that more flow was disturbed if an incident occurs earlier during the evacuation, which leads to a greater amount of time to reroute the affected flow and clear the system. When the time for plan updates was delayed (i.e., the rerouting process takes place later), the clearance time of the network increased accordingly. This emphasizes the importance of making timely decisions for fast response to incidents as it is crucial in an efficient evacuation rerouting plan. The proposed approach has also been tested on a large-scale evacuation network, and the results support that the proposed approach can handle in evacuating large metropolitan areas in a timely manner.

The proposed approach is limited to deterministic problem settings. As a future work, one can extend this paper by considering various uncertainties such as variations in the number of would-be evacuees, alternative road capacities, or evacuees' behavior. Another venue for an extension is to extend the proposed approach to make it as a basis for vulnerabilities analysis through developing a probabilistic mechanism that accounts for factors including simultaneous multiple occurrences.

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Appendix A

Path-Based Model (Lim et al. 2012): The path-based model (*PBM*) for the evacuation planning problem is described below (18)-(22). The objective function of the program minimizes the sum of unmet demand β_n of all source nodes $n \in \mathcal{N}_o$. Constraint (19) guarantees that the total flow that exits a source node $n \in \mathcal{N}_o$ plus the remaining flow on the source node (unmet demand) is greater than the demand on the source node.

Constraint (20) places a restriction on the total amount of flow that reaches arc $a \in \mathcal{A}$ at time $t \in \mathbb{T}$ and ensures that it is less than the capacity of the road C_a. Parameter δ_{pa} has binary values and takes value 1 only if arc $a \in \mathcal{A}$ belongs to path $p \in \wp$. Variable $f_{p(t-\theta_{pa})}$ showcases the flow that exits the source node of path $p \in \wp$ at time $t - \theta_{pa}$. Since θ_{pa} represents the transit time of the flow from the origin of the path to arc $a \in \mathcal{A}$, the flow reaches arc $a \in \mathcal{A}$ at time $t \in \mathbb{T}$. The total flow that reaches a destination node is restricted by the capacity of the node. Non-negativity and integrality conditions of the variables are shown in Constraints (22) and (23).

$$\text{Minimize} \quad \sum_{n \in \mathcal{N}_o} \beta_n \qquad \textit{(PBM)} \tag{18}$$

$$\text{Subject to:} \quad \sum_{p \in \wp_n^+} \sum_{t \in \mathbb{T}} f_{pt} + \beta_n \geq D_n, \qquad \forall n \in \mathcal{N}_o, \tag{19}$$

$$\sum_{p \in \wp} \delta_{pa} f_{p(t-\theta_{pa})} \leq C_a, \qquad \forall a \in \mathcal{A},\ \forall t \in \mathbb{T}, \tag{20}$$

$$\sum_{p \in \wp} \sum_{t \in \mathbb{T}} K_{pn}\, f_{pt} \leq \ell_n, \qquad \forall n \in \mathcal{N}_d, \tag{21}$$

$$f_{pt} \in \mathbb{Z}^+ \qquad \forall p \in \wp, \forall t \in \mathbb{T}, \tag{22}$$

$$\beta_n \in \mathbb{Z}^+ \qquad \forall n \in \mathcal{N}_o \tag{23}$$

Appendix B

This section is to explain our motivation for the proposed evacuation reroute planning model formulation. The pre-processing algorithms help us to develop a less complex optimization model which can be solved in a timely manner for the application discussed in this paper. Let us explain what happens if we were going to solve the optimization without the pre-processing algorithms. The main challenge lies in calculating the amount of disturbed flow (H_{pnt}). If the proposed pre-processing algorithms are not used, the following constraint should be included in the optimization framework to calculate the amount of disturbed flow (H_{pnt}) on node $n \in \mathcal{N}$ of path $p \in \wp$ at time $t \in \mathbb{T}$.

$$H_{np(t+\acute{\theta}_{pn})} \geq f_{pt}\gamma_{na}\delta_{pa}\frac{\left|t+\theta_{pa}+\tau_a-DT_a\right|}{t+\theta_{pa}+\tau_a-DT_a} - \left(\sum_{\acute{n}\in N_{pn}}\gamma_{n\acute{a}}\delta_{p\acute{a}}\frac{\left|t+\theta_{p\acute{a}}+\tau_{\acute{a}}-DT_{\acute{a}}\right|}{t+\theta_{p\acute{a}}+\tau_{\acute{a}}-DT_{\acute{a}}}+\left|N_{pn}\right|\right)M \qquad \forall p \in \wp, n \in \mathcal{N}, t \in \mathbb{T},$$

where M is an arbitrarily large number and N_{pn} is the set of all preceding nodes to node $n \in \mathcal{N}$ of path $p \in \wp$. Parameters γ_{na} and δ_{pa} are used to represent the topology of the network (see the notation in p. 5). Flow f_{pt} departs from the origin of path $p \in \wp$ at time $t \in \mathbb{T}$ and reaches the end of arc $a \in \mathcal{A}$ at time $t + \theta_{pa} + \tau_a$. The term $\left|t+\theta_{pa}+\tau_a-DT_a\right|/t+\theta_{pa}+\tau_a-DT_a$ is used to monitor arc disruption time DT_a on the flow, f_{pt}. If a disruption on the arc occurs after the flow has passed the incident arc, then $t+\theta_{pa}+\tau_a < DT_a$; hence, $\left|t+\theta_{pa}+\tau_a-DT_a\right|/t+\theta_{pa}+\tau_a-DT_a = -1$. If the disruption happens before the flow arrives at the end of the arc, then we will have $\left|t+\theta_{pa}+\tau_a-DT_a\right|/t+\theta_{pa}+\tau_a-DT_a = 1$. To indicate whether flow f_{pt} is disturbed by arc $a \in \mathcal{A}$, we also need to take into account the effect of disruption times of the preceding arcs to arc $a \in \mathcal{A}$. If the flow is not interrupted by any preceding arcs to the incident arc, we will have $\sum_{\acute{n}\in N_{pn}}\gamma_{n\acute{a}}\delta_{p\acute{a}}\left|t+\theta_{p\acute{a}}+\tau_{\acute{a}}-DT_{\acute{a}}\right|/t+\theta_{p\acute{a}}+\tau_{\acute{a}}-DT_{\acute{a}} = -\left|N_{pn}\right|$. Hence, if flow f_{pt} is stopped due to the disruption on arc $a \in \mathcal{A}$ (which emerges from node $n \in \mathcal{N}$) and not any other preceding arcs, the following equations hold:

$$\left|t+\theta_{pa}+\tau_a-DT_a\right|/t+\theta_{pa}+\tau_a-DT_a = 1$$

$$\sum_{\acute{n}\in N_{pn}}\gamma_{n\acute{a}}\delta_{p\acute{a}}\left|t+\theta_{p\acute{a}}+\tau_{\acute{a}}-DT_{\acute{a}}\right|/t+\theta_{p\acute{a}}+\tau_{\acute{a}}-DT_{\acute{a}} = -\left|N_{pn}\right|$$

This leads to $H_{np(t+\acute{\theta}_{pn})} = f_{pt}$ if the objective function is minimized, i.e., the amount of disturbed flow is minimized. For any other scenarios, the constraint can be relaxed with an exception when the disruption occurrence time equals the time that the flow arrives at the end of the arc ($DT_a = t + \theta_{pa} + \tau_a$). In this case, we will have the following expression $\frac{0}{0}$ in the constraint, which cannot be not defined.

To overcome this modeling complexity, we proposed to calculate the values of parameter V_{pnt} *a priori* using the pre-processing algorithms. As a result, it simplifies the equation $H_{pn(t+\acute{\theta}_{pn})} = V_{pnt}f_{pt}$ in the *RPBM* as V_{pnt} is not an unknown variable, but a parameter.

Appendix C

Table 7: Node sequence of candidate paths

Paths	Sequences	Paths	Sequences
P1	1-4-7-9	**P22**	2-4-5-8-10
P2	1-4-6-7-9	**P23**	2-4-7-8-10
P3	1-4-6-8-10	**P24**	2-4-6-7-8-10
P4	1-4-5-6-7-9	**P25**	2-4-6-8-7-9
P5	1-4-5-6-8-10	**P26**	2-4-8-10
P6	1-4-5-8-10	**P27**	2-5-4-7-9
P7	1-4-7-8-10	**P28**	3-5-6-7-9
P8	1-4-6-7-8-10	**P29**	3-5-6-8-10
P9	1-4-6-8-7-9	**P30**	3-5-8-10
P10	1-4-8-10	**P31**	3-4-7-9
P11	1-5-6-7-9	**P32**	3-4-6-7-9
P12	1-5-6-8-10	**P33**	3-4-6-8-10
P13	1-5-8-10	**P34**	3-5-4-7-9
P14	2-4-7-9	**P35**	3-5-4-6-7-9
P15	2-4-6-7-9	**P36**	3-5-4-6-8-10
P16	2-4-6-8-10	**P37**	3-5-7-9
P17	2-5-6-7-9	**P38**	3-5-6-7-8-10
P18	2-5-6-8-10	**P39**	3-5-6-8-7-9
P19	2-5-8-10	**P40**	3-5-8-7-9
P20	2-4-5-6-7-9	**P41**	3-4-5-6-7-9
P21	2-4-5-6-8-10	**P42**	3-4-5-6-8-10